

# Degenerations of Kählerian K3 surfaces with finite symplectic automorphism groups. III\*

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## Abstract

Similarly to our papers [19], [20], we classify degenerations of codimension 2 and higher of Kählerian K3 surfaces with finite symplectic automorphism groups. In [19], [20], it was done for the codimension one.

## 1 Introduction

Similarly to our papers [19], [20], we classify degenerations of codimension 2 and higher of Kählerian K3 surfaces with finite symplectic automorphism groups. In [19], [20], it was done for the codimension 1. For readers convenience, in Section 3, we give the Table 1 from [20] which gives the main classification of such degenerations of codimension 1.

In this variant of the paper, we do classification of degenerations of codimension  $\geq 2$  for finite symplectic automorphism groups of order  $\geq 6$ .

For groups of order  $> 8$  and the group  $Q_8$  of order 8, it is done in Section 3. It has Table 2 which contains classification of types and lattices  $S$  of such degenerations. They are main invariants of degenerations. Table 3 of Section 3 gives markings of such degenerations by Niemeier lattices  $N_j$  in notations of [18], [19] and [20]. For such groups, the codimension of degenerations is less or equal to 4, and for the most of them it is less or equal to 3.

We consider remaining difficult groups  $D_8$  and  $(C_2)^3$  of order 8 in Sections 4 and 5. There are too many cases for these difficult groups, and it is better to consider them separately.

We hope to consider remaining groups  $D_6$ ,  $C_4$ ,  $(C_2)^2$ ,  $C_3$ ,  $C_2$  and give more results and details in further variants of the paper and further publications.

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\*With support by Russian Scientific Fund N 14-50-00005.

## 2 Classification of degenerations of Kählerian K3 surfaces with finite symplectic automorphism groups. The main theory.

Let  $X$  be a Kählerian K3 surface (e. g. see [22], [21], [2], [23], [24] about such surfaces). That is  $X$  is a non-singular compact complex surface with the trivial canonical class  $K_X$ , and its irregularity  $q(X)$  is equal to 0. Then  $H^2(X, \mathbb{Z})$  with the intersection pairing is an even unimodular lattice  $L_{K3}$  of the signature  $(3, 19)$ . The primitive sublattice  $S_X = H^2(X, \mathbb{Z}) \cap H^{1,1}(X) \subset H^2(X, \mathbb{Z})$  is the *Picard lattice* of  $X$  generated by first Chern classes of all line bundles over  $X$ .

Let  $G$  be a finite symplectic automorphism group of  $X$ . Here symplectic means that for any  $g \in G$ , for a non-zero holomorphic 2-form  $\omega_X \in H^{2,0}(X) = \Omega^2[X] = \mathbb{C}\omega_X$ , one has  $g^*(\omega_X) = \omega_X$ . For an  $G$ -invariant sublattice  $M \subset H^2(X, \mathbb{Z})$ , we denote by  $M^G = \{x \in M \mid G(x) = x\}$  the *invariant sublattice* of  $M$ , and by  $M_G = (M^G)^\perp_M$  the *coinvariant sublattice* of  $M$ . By [13], the coinvariant lattice  $S_G = H^2(X, \mathbb{Z})_G = (S_X)_G$  is *Leech type lattice*: i. e. it is negative definite, it has no elements with square  $(-2)$ ,  $G$  acts trivially on the discriminant group  $As_G = (S_G)^*/S_G$ , and  $(S_G)^G = \{0\}$ . For a general pair  $(X, G)$ , the  $S_G = S_X$ , and non-general  $(X, G)$  can be considered as Kählerian K3 surfaces with the condition  $S_G \subset S_X$  on the Picard lattice (in terminology of [13]). The dimension of their moduli is equal to  $20 - \text{rk } S_G$ .

Let  $E \subset X$  be a non-singular irreducible rational curve (that is  $E \cong \mathbb{P}^1$ ). It is equivalent to:  $\alpha = cl(E) \in S_X$ ,  $\alpha^2 = -2$ ,  $\alpha$  is effective and  $\alpha$  is numerically effective:  $\alpha \cdot D \geq 0$  for every irreducible curve  $D$  on  $X$  such that  $cl(D) \neq \alpha$ .

Let us consider the primitive sublattice  $S = [S_G, G(\alpha)]_{pr} \subset S_X$  of  $S_X$  generated by the coinvariant sublattice  $S_G$  and all classes of the orbit  $G(E)$ . We remind that primitive means that  $S_X/S$  has not torsion. Since  $S_G$  has no elements with square  $(-2)$ , it follows that  $\text{rk } S = \text{rk } S_G + 1$  and  $S = [S_G, \alpha]_{pr} \subset S_X$ .

Let us *assume that the lattice*  $S = [S_G, \alpha]_{pr}$  *is negative definite*. Then the elements  $G(\alpha)$  define the basis of the root system  $\Delta(S)$  of all elements with square  $(-2)$  of  $S$ . All curves  $G(E)$  of  $X$  can be contracted to Du Val singularities of types of connected components of the Dynkin diagram of the basis. The group  $G$  will act on the corresponding singular K3 surface  $\bar{X}$  with these Du Val singularities. For a general such triplet  $(X, G, G(E))$ , the Picard lattice  $S_X = S$ , and such triplets can be considered as *a degeneration of codimension 1* of Kählerian K3 surfaces  $(X, G)$  with the finite symplectic automorphism group  $G$ . Really, the dimension of moduli of Kählerian K3 surfaces with the condition  $S \subset S_X$  on the Picard lattice is equal to  $20 - \text{rk } S = 20 - \text{rk } S_G - 1$ .

By Global Torelli Theorem for K3 surfaces [21], [2], the main invariants of the degenera-

tion is the *type of the abstract group*  $G$  which is equivalent to the isomorphism class of the coinvariant lattice  $S_G$ , and the type of the degeneration which is equivalent to the Dynkin diagram of the basis  $G(\alpha)$  or the Dynkin diagram of the rational curves  $G(E)$ .

We can consider only the maximal finite symplectic automorphism group  $G$  with the same coinvariant lattice  $S_G$ , that is  $G = Clos(G)$ . By Global Torelli Theorem for K3 surfaces, this is equivalent to

$$G|_{S_G} = \{g \in O(S_G) \mid g \text{ is identity on } A_{S_G} = (S_G)^*/S_G\}.$$

Indeed,  $G$  and  $Clos(G)$  have the same lattice  $S_G$ , the same orbits  $G(E)$  and  $Clos(G)(E)$ , and the same sublattice  $S \subset S_X$ .

In [19], all types of  $G = Clos(G)$  and types of degenerations (that is Dynkin diagrams of the orbits  $G(E)$ ) are described. They are described in Table 1 below which is the same as [20, Table 1] where  $\mathbf{n}$  gives types of possible  $G = Clos(G)$ , and we show all possible types of degenerations at the corresponding rows by their Dynkin diagrams.

In Table 1, the type of  $G = Clos(G)$  and the isomorphism class of the lattice  $S_G$  is marked by  $\mathbf{n}$ . We also give the genus of  $S_G$  which is defined by the discriminant quadratic form  $q_{S_G}$ . They were calculated in papers [14], [11], [25], [5].

In [20], with two exceptions which are marked by *I* and *II* for  $\mathbf{n} = 10$  (group  $D_8$ ) and the degeneration  $2A_1$ , and for  $\mathbf{n} = 34$  (group  $\mathfrak{S}_4$ ) and the degeneration  $6A_1$ , it was shown that the lattice  $S = [S_G, \alpha]_{pr}$  is unique, up to isomorphisms, and its genus (equivalent to  $\text{rk } S$  and the discriminant quadratic form  $q_S$ ) is shown in Table 1. In [20, Table 2], we described possible markings of  $S$ ,  $G$  and  $\alpha$  by Niemeier lattices which give exact lattices descriptions of  $S$ , and the action of  $G$  on  $S$  and  $G(\alpha)$ .

For degenerations of arbitrary codimension  $t \geq 1$  which we consider in this paper, instead of one orbit  $G(E)$  of a non-singular rational curve, we should consider  $t \geq 1$  different orbits  $G(E_1), \dots, G(E_k)$  of non-singular rational curves on  $X$ , and their classes  $G(\alpha_1), \dots, G(\alpha_k)$  in  $S_X$ , but we also assume that the sublattice  $S = [S_G, G(\alpha_1), \dots, G(\alpha_k)]_{pr} \subset S_X$  is negative definite. Then the codimension of the degeneration is equal to  $t = \text{rk } S - \text{rk } S_G$ , and  $S_X = S = [S_G, \alpha_1, \dots, \alpha_t]_{pr}$  in general. We remark that  $\text{rk } S = \text{rk } S_G + t \leq 19$  since  $H^2(X, \mathbb{Z})$  has the signature  $(3, 19)$ . Thus,  $t \leq 19 - \text{rk } S_G$ .

The type of the degeneration is given by the Dynkin diagrams and subdiagrams

$$(Dyn(G(\alpha_1)), \dots, Dyn(G(\alpha_t))) \subset Dyn(G(\alpha_1) \cup \dots \cup G(\alpha_t))$$

and their types. In difficult cases, we also consider the matrix of subdiagrams which is defined by

$$(Dyn(G(\alpha_i)), Dyn(G(\alpha_j))) \subset Dyn(G(\alpha_i) \cup G(\alpha_j))$$

and their types for  $1 \leq i < j \leq t$ .

In Table 2 which is similar to Table 1 for codimension 1, we give classification of types of degenerations of arbitrary codimension  $\geq 2$  for  $\mathbf{n} \geq 12$  (then the codimension is less or equal to 4). Equivalently, either  $|G| > 8$  or  $G \cong Q_8$  of order 8 ( $G$  is big enough). We calculate genres of the lattices  $S$  by  $\text{rk } S$  and  $q_S$ . By  $*$ , we mark cases when we prove that the lattice  $S$  is unique up to isomorphisms, for the given type. In Table 3 which is similar to [20, Table 2] for the codimension 1, we give the description of their markings  $S \subset N_j$  by Niemeier lattices  $N_j$ ,  $j = 1, \dots, 24$ , where we use results and notations of [17], [18] and [19]. For remaining groups  $D_8$  and  $(C_2)^3$  of order 8, similar results are given in Sections 4 and 5.

All calculations are similar to our calculations in [19] and [20] for codimension 1, and they use similar to [19] and [20] computer programs.

We hope to give more details and other results in further variants of the paper and further publications.

### 3 Types of degenerations for $\mathbf{n} \geq 12$

We remind that Niemeier lattices are negative definite even unimodular lattices of the rank 24. There are 24 such lattices, up to isomorphisms,  $N = N_j$ ,  $j = 1, 2, \dots, 24$ , classified by Niemeier. They are characterized by their root sublattices  $N^{(2)}$  generated by all their elements with square  $(-2)$  (called roots). Further,  $\Delta(N)$  is the set of all roots of  $N$ . We have the following list of Niemeier lattices  $N_j$  where the number  $j$  is shown in the bracket:

$$N^{(2)} = [\Delta(N)] =$$

- (1)  $D_{24}$ , (2)  $D_{16} \oplus E_8$ , (3)  $3E_8$ , (4)  $A_{24}$ , (5)  $2D_{12}$ , (6)  $A_{17} \oplus E_7$ , (7)  $D_{10} \oplus 2E_7$ ,
- (8)  $A_{15} \oplus D_9$ , (9)  $3D_8$ , (10)  $2A_{12}$ , (11)  $A_{11} \oplus D_7 \oplus E_6$ , (12)  $4E_6$ , (13)  $2A_9 \oplus D_6$ ,
- (14)  $4D_6$ , (15)  $3A_8$ , (16)  $2A_7 \oplus 2D_5$ , (17)  $4A_6$ , (18)  $4A_5 \oplus D_4$ , (19)  $6D_4$ ,
- (20)  $6A_4$ , (21)  $8A_3$ , (22)  $12A_2$ , (23)  $24A_1$

give 23 Niemeier lattices  $N_j$ . The last is Leech lattice (24) with  $N^{(2)} = \{0\}$  which has no roots.

Table 1: Types and lattices  $S$  of degenerations of codimension 1 of Kählerian K3 surfaces with finite symplectic automorphism groups  $G = Clos(G)$ .

$\mathbf{n}$	$ G $	$i$	$G$	$\text{rk } S_G$	$q_{S_G}$	$Deg$	$\text{rk } S$	$q_S$
1	2	1	$C_2$	8	$2_{II}^{+8}$	$\mathbb{A}_1$	9	$2_7^{+9}$
						$2\mathbb{A}_1$	9	$2_{II}^{-6}, 4_3^{-1}$
2	3	1	$C_3$	12	$3^{+6}$	$\mathbb{A}_1$	13	$2_3^{-1}, 3^{+6}$
						$3\mathbb{A}_1$	13	$2_1^{+1}, 3^{-5}$
3	4	2	$C_2^2$	12	$2_{II}^{-6}, 4_{II}^{-2}$	$\mathbb{A}_1$	13	$2_3^{+7}, 4_{II}^{+2}$
						$2\mathbb{A}_1$	13	$2_{II}^{-4}, 4_7^{-3}$
						$4\mathbb{A}_1$	13	$2_{II}^{-6}, 8_3^{-1}$
4	4	1	$C_4$	14	$2_2^{+2}, 4_{II}^{+4}$	$\mathbb{A}_1$	15	$2_5^{-3}, 4_{II}^{+4}$
						$2\mathbb{A}_1$	15	$4_1^{-5}$
						$4\mathbb{A}_1$	15	$2_2^{+2}, 4_{II}^{+2}, 8_7^{+1}$
						$\mathbb{A}_2$	15	$2_1^{+1}, 4_{II}^{-4}$
6	6	1	$D_6$	14	$2_{II}^{-2}, 3^{+5}$	$\mathbb{A}_1$	15	$2_7^{-3}, 3^{+5}$
						$2\mathbb{A}_1$	15	$4_3^{-1}, 3^{+5}$
						$3\mathbb{A}_1$	15	$2_1^{-3}, 3^{-4}$
						$6\mathbb{A}_1$	15	$4_1^{+1}, 3^{+4}$
9	8	5	$C_2^3$	14	$2_{II}^{+6}, 4_2^{+2}$	$2\mathbb{A}_1$	15	$2_{II}^{-4}, 4_5^{-3}$
						$4\mathbb{A}_1$	15	$2_{II}^{+6}, 8_1^{+1}$
						$8\mathbb{A}_1$	15	$2_{II}^{+6}, 4_1^{+1}$
10	8	3	$D_8$	15	$4_1^{+5}$	$\mathbb{A}_1$	16	$2_1^{+1}, 4_7^{+5}$
						$(2\mathbb{A}_1)_I$	16	$2_6^{-2}, 4_6^{-4}$
						$(2\mathbb{A}_1)_{II}$	16	$2_{II}^{+2}, 4_{II}^{+4}$
						$4\mathbb{A}_1$	16	$4_7^{+3}, 8_1^{+1}$
						$8\mathbb{A}_1$	16	$4_0^{+4}$
						$2\mathbb{A}_2$	16	$4_{II}^{+4}$
12	8	4	$Q_8$	17	$2_7^{-3}, 8_{II}^{-2}$	$8\mathbb{A}_1$	18	$2_7^{-3}, 16_3^{-1}$
						$\mathbb{A}_2$	18	$2_6^{-2}, 8_{II}^{-2}$
16	10	1	$D_{10}$	16	$5^{+4}$	$\mathbb{A}_1$	17	$2_7^{+1}, 5^{+4}$
						$5\mathbb{A}_1$	17	$2_7^{+1}, 5^{-3}$

<b>n</b>	$ G $	$i$	$G$	$\text{rk } S_G$	$q_{S_G}$	$Deg$	$\text{rk } S$	$q_S$
17	12	3	$\mathfrak{A}_4$	16	$2_{II}^{-2}, 4_{II}^{-2}, 3^{+2}$	$\mathbb{A}_1$	17	$2_7^{-3}, 4_{II}^{+2}, 3^{+2}$
						$3\mathbb{A}_1$	17	$2_1^{-3}, 4_{II}^{+2}, 3^{-1}$
						$4\mathbb{A}_1$	17	$2_{II}^{-2}, 8_3^{-1}, 3^{+2}$
						$6\mathbb{A}_1$	17	$4_1^{-3}, 3^{+1}$
						$12\mathbb{A}_1$	17	$2_{II}^{-2}, 8_1^{+1}, 3^{-1}$
18	12	4	$D_{12}$	16	$2_{II}^{+4}, 3^{+4}$	$\mathbb{A}_1$	17	$2_7^{+5}, 3^{+4}$
						$2\mathbb{A}_1$	17	$2_{II}^{+2}, 4_7^{+1}, 3^{+4}$
						$3\mathbb{A}_1$	17	$2_5^{-5}, 3^{-3}$
						$6\mathbb{A}_1$	17	$2_{II}^{-2}, 4_1^{+1}, 3^{+3}$
21	16	14	$C_2^4$	15	$2_{II}^{+6}, 8_I^{+1}$	$4\mathbb{A}_1$	16	$2_{II}^{+4}, 4_{II}^{+2}$
						$16\mathbb{A}_1$	16	$2_{II}^{+6}$
22	16	11	$C_2 \times D_8$	16	$2_{II}^{+2}, 4_0^{+4}$	$2\mathbb{A}_1$	17	$4_7^{+5}$
						$4\mathbb{A}_1$	17	$2_{II}^{+2}, 4_0^{+2}, 8_7^{+1}$
						$8\mathbb{A}_1$	17	$2_{II}^{+2}, 4_7^{+3}$
26	16	8	$SD_{16}$	18	$2_7^{+1}, 4_7^{+1}, 8_{II}^{+2}$	$8\mathbb{A}_1$	19	$2_1^{+1}, 4_1^{+1}, 16_3^{-1}$
						$2\mathbb{A}_2$	19	$2_5^{-1}, 8_{II}^{-2}$
30	18	4	$\mathfrak{A}_{3,3}$	16	$3^{+4}, 9^{-1}$	$3\mathbb{A}_1$	17	$2_5^{-1}, 3^{-3}, 9^{-1}$
						$9\mathbb{A}_1$	17	$2_3^{-1}, 3^{+4}$
32	20	3	$Hol(C_5)$	18	$2_6^{-2}, 5^{+3}$	$2\mathbb{A}_1$	19	$4_1^{+1}, 5^{+3}$
						$5\mathbb{A}_1$	19	$2_1^{+3}, 5^{-2}$
						$10\mathbb{A}_1$	19	$4_5^{-1}, 5^{+2}$
						$5\mathbb{A}_2$	19	$2_5^{-1}, 5^{-2}$
33	21	1	$C_7 \rtimes C_3$	18	$7^{+3}$	$7\mathbb{A}_1$	19	$2_1^{+1}, 7^{+2}$
34	24	12	$\mathfrak{S}_4$	17	$4_3^{+3}, 3^{+2}$	$\mathbb{A}_1$	18	$2_5^{-1}, 4_1^{+3}, 3^{+2}$
						$2\mathbb{A}_1$	18	$2_2^{+2}, 4_{II}^{+2}, 3^{+2}$
						$3\mathbb{A}_1$	18	$2_7^{+1}, 4_5^{-3}, 3^{-1}$
						$4\mathbb{A}_1$	18	$4_3^{-1}, 8_3^{-1}, 3^{+2}$
						$(6\mathbb{A}_1)_I$	18	$2_4^{-2}, 4_0^{+2}, 3^{+1}$
						$(6\mathbb{A}_1)_{II}$	18	$2_{II}^{+2}, 4_{II}^{-2}, 3^{+1}$
						$8\mathbb{A}_1$	18	$4_2^{+2}, 3^{+2}$
						$12\mathbb{A}_1$	18	$4_5^{-1}, 8_7^{+1}, 3^{-1}$
						$6\mathbb{A}_2$	18	$4_{II}^{-2}, 3^{+1}$

<b>n</b>	$ G $	$i$	$G$	$\text{rk } S_G$	$q_{S_G}$	$Deg$	$\text{rk } S$	$q_S$
39	32	27	$2^4C_2$	17	$2_{II}^{+2}, 4_0^{+2}, 8_7^{+1}$	$4\mathbb{A}_1$	18	$4_6^{+4}$
						$8\mathbb{A}_1$	18	$2_{II}^{+2}, 4_7^{+1}, 8_7^{+1}$
						$16\mathbb{A}_1$	18	$2_{II}^{+2}, 4_6^{+2}$
40	32	49	$Q_8 * Q_8$	17	$4_7^{+5}$	$8\mathbb{A}_1$	18	$4_6^{+4}$
46	36	9	$3^2C_4$	18	$2_6^{-2}, 3^{+2}, 9^{-1}$	$6\mathbb{A}_1$	19	$4_7^{+1}, 3^{+1}, 9^{-1}$
						$9\mathbb{A}_1$	19	$2_5^{-3}, 3^{+2}$
						$9\mathbb{A}_2$	19	$2_5^{-1}, 3^{+2}$
48	36	10	$\mathfrak{S}_{3,3}$	18	$2_{II}^{-2}, 3^{+3}, 9^{-1}$	$3\mathbb{A}_1$	19	$2_5^{+3}, 3^{-2}, 9^{-1}$
						$6\mathbb{A}_1$	19	$4_1^{+1}, 3^{+2}, 9^{-1}$
						$9\mathbb{A}_1$	19	$2_7^{-3}, 3^{+3}$
49	48	50	$2^4C_3$	17	$2_{II}^{-4}, 8_1^{+1}, 3^{-1}$	$4\mathbb{A}_1$	18	$2_{II}^{-2}, 4_{II}^{+2}, 3^{-1}$
						$12\mathbb{A}_1$	18	$2_{II}^{-2}, 4_2^{-2}$
						$16\mathbb{A}_1$	18	$2_{II}^{-4}, 3^{-1}$
51	48	48	$C_2 \times \mathfrak{S}_4$	18	$2_{II}^{+2}, 4_2^{+2}, 3^{+2}$	$2\mathbb{A}_1$	19	$4_1^{+3}, 3^{+2}$
						$4\mathbb{A}_1$	19	$2_{II}^{+2}, 8_1^{+1}, 3^{+2}$
						$6\mathbb{A}_1$	19	$4_7^{-3}, 3^{+1}$
						$8\mathbb{A}_1$	19	$2_{II}^{-2}, 4_5^{-1}, 3^{+2}$
						$12\mathbb{A}_1$	19	$2_{II}^{-2}, 8_7^{+1}, 3^{-1}$
55	60	5	$\mathfrak{A}_5$	18	$2_{II}^{-2}, 3^{+1}, 5^{-2}$	$\mathbb{A}_1$	19	$2_7^{-3}, 3^{+1}, 5^{-2}$
						$5\mathbb{A}_1$	19	$2_3^{+3}, 3^{+1}, 5^{+1}$
						$6\mathbb{A}_1$	19	$4_1^{+1}, 5^{-2}$
						$10\mathbb{A}_1$	19	$4_7^{+1}, 3^{+1}, 5^{-1}$
						$15\mathbb{A}_1$	19	$2_5^{+3}, 5^{-1}$
56	64	138	$\Gamma_{25}a_1$	18	$4_5^{+3}, 8_1^{+1}$	$8\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1}$
						$16\mathbb{A}_1$	19	$4_5^{+3}$
61	72	43	$\mathfrak{A}_{4,3}$	18	$4_{II}^{-2}, 3^{-3}$	$3\mathbb{A}_1$	19	$2_5^{-1}, 4_{II}^{+2}, 3^{+2}$
						$12\mathbb{A}_1$	19	$8_1^{+1}, 3^{+2}$
65	96	227	$2^4D_6$	18	$2_{II}^{-2}, 4_7^{+1}, 8_1^{+1}, 3^{-1}$	$4\mathbb{A}_1$	19	$4_3^{-3}, 3^{-1}$
						$8\mathbb{A}_1$	19	$2_{II}^{-2}, 8_7^{+1}, 3^{-1}$
						$12\mathbb{A}_1$	19	$4_5^{+3}$
						$16\mathbb{A}_1$	19	$2_{II}^{+2}, 4_3^{-1}, 3^{-1}$
75	192	1023	$4^2\mathfrak{A}_4$	18	$2_{II}^{-2}, 8_6^{-2}$	$16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1}$

Table 2: Types and lattices  $S$  of degenerations of codimension  $\geq 2$  of Kählerian K3 surfaces with finite symplectic automorphism groups  $G = \text{Clos}(G)$  for  $\mathbf{n} \geq 12$ .

$\mathbf{n}$	$ G $	$i$	$G$	$\text{rk } S_G$	$Deg$	$\text{rk } S$	$q_S$
12	8	4	$Q_8$	17	$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1} *$
					$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 8\mathbb{A}_2$	19	$2_7^{-3}, 3^{-1} *$
					$(8\mathbb{A}_1, \mathbb{A}_2) \subset 8\mathbb{A}_1 \amalg \mathbb{A}_2$	19	$2_2^{+2}, 16_3^{-1} *$
					$(\mathbb{A}_2, \mathbb{A}_2) \subset 2\mathbb{A}_2$	19	$2_5^{-1}, 8_{II}^{-2} *$
16	10	1	$D_{10}$	16	$(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$	18	$2_6^{+2}, 5^{+4}$
					$(\mathbb{A}_1, 5\mathbb{A}_1) \subset 6\mathbb{A}_1$	18	$2_6^{+2}, 5^{-3}$
					$(5\mathbb{A}_1, 5\mathbb{A}_1) \subset 10\mathbb{A}_1$	18	$2_6^{+2}, 5^{+2} *$
					$(5\mathbb{A}_1, 5\mathbb{A}_1) \subset 5\mathbb{A}_2$	18	$3^{-1}, 5^{-2} *$
					$(\mathbb{A}_1, \mathbb{A}_1, 5\mathbb{A}_1) \subset 7\mathbb{A}_1$	19	$2_5^{+3}, 5^{-3}$
					$(\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 11\mathbb{A}_1$	19	$2_5^{+3}, 5^{+2}$
					$(\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 5\mathbb{A}_2$	19	$2_7^{+1}, 3^{-1}, 5^{-2}$
					$(5\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 15\mathbb{A}_1$	19	$2_1^{-3}, 5^{-1} *$
					$(5\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 5\mathbb{A}_2 \amalg 5\mathbb{A}_1$	19	$2_7^{+1}, 3^{-1}, 5^{+1} *$
					$\begin{pmatrix} 5\mathbb{A}_1 & 5\mathbb{A}_2 & 10\mathbb{A}_1 \\ & 5\mathbb{A}_1 & 5\mathbb{A}_2 \\ & & 5\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_3$	19	$4_1^{+1}, 5^{+1} *$
17	12	3	$\mathfrak{A}_4$	16	$(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$	18	$2_2^{+2}, 4_{II}^{+2}, 3^{+2} *$
					$(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$	18	$2_{II}^{-2}, 4_{II}^{+2}, 3^{-1} *$
					$(\mathbb{A}_1, 4\mathbb{A}_1) \subset 5\mathbb{A}_1$	18	$2_3^{+3}, 8_7^{+1}, 3^{+2} *$
					$(\mathbb{A}_1, 6\mathbb{A}_1) \subset 7\mathbb{A}_1$	18	$2_1^{+1}, 4_7^{+3}, 3^{+1} *$
					$(\mathbb{A}_1, 12\mathbb{A}_1) \subset 13\mathbb{A}_1$	18	$2_3^{+3}, 8_1^{+1}, 3^{-1} *$
					$(3\mathbb{A}_1, 4\mathbb{A}_1) \subset 7\mathbb{A}_1$	18	$2_5^{+3}, 8_7^{+1}, 3^{-1} *$
					$(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_3$	18	$2_6^{+2}, 4_{II}^{-2} *$
					$(3\mathbb{A}_1, 12\mathbb{A}_1) \subset 15\mathbb{A}_1$	18	$2_1^{-3}, 8_1^{+1} *$
					$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	18	$4_2^{+2}, 3^{+2} *$
					$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 4\mathbb{A}_2$	18	$2_{II}^{-2}, 3^{+3} *$
					$(4\mathbb{A}_1, 6\mathbb{A}_1) \subset 10\mathbb{A}_1$	18	$4_1^{+1}, 8_3^{-1}, 3^{+1} *$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$	18	$2_{II}^{-4}, 3^{-1} *$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 4\mathbb{D}_4$	18	$2_{II}^{-2}, 3^{-1} *$
					$(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$2_{II}^{-2}, 4_2^{-2} *$



					$(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 6\mathbb{A}_2$	18	$4_{II}^{-2}, 3^{+1} *$
					$(6\mathbb{A}_1, 12\mathbb{A}_1) \subset 6\mathbb{A}_3$	18	$4_6^{+2} *$
					$(\mathbb{A}_1, \mathbb{A}_1, \mathbb{A}_1) \subset 3\mathbb{A}_1$	19	$2_1^{+1}, 4_{II}^{+2}, 3^{+2} *$
					$(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$	19	$2_2^{+2}, 8_7^{+1}, 3^{+2} *$
					$(\mathbb{A}_1, \mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1$	19	$4_7^{-3}, 3^{+1} *$
					$(\mathbb{A}_1, \mathbb{A}_1, 12\mathbb{A}_1) \subset 14\mathbb{A}_1$	19	$2_2^{+2}, 8_5^{-1}, 3^{-1} *$
					$(\mathbb{A}_1, 3\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	19	$2_{II}^{-2}, 8_7^{+1}, 3^{-1} *$
					$(\mathbb{A}_1, 3\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1} *$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 9\mathbb{A}_1$	19	$2_1^{+1}, 4_0^{+2}, 3^{+2} *$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_7^{-3}, 3^{+3} *$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 11\mathbb{A}_1$	19	$2_3^{-1}, 4_7^{+1}, 8_5^{-1}, 3^{+1}$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 12\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{D}_4$	19	$2_7^{-3}, 3^{-1} *$
					$(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$2_3^{-1}, 4_{II}^{-2}, 3^{+1} *$
					$(\mathbb{A}_1, 6\mathbb{A}_1, 12\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_3$	19	$2_7^{+1}, 4_6^{+2}$
					$(3\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 11\mathbb{A}_1$	19	$2_1^{+1}, 4_2^{+2}, 3^{-1} *$
					$(3\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_5^{+3}, 3^{-2} *$
					$(3\mathbb{A}_1, 4\mathbb{A}_1, 12\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 4\mathbb{D}_4$	19	$2_5^{+3} *$
					$(3\mathbb{A}_1, 6\mathbb{A}_1, 4\mathbb{A}_1) \subset 3\mathbb{A}_3 \amalg 4\mathbb{A}_1$	19	$2_6^{+2}, 8_3^{-1}$
					$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$	19	$8_1^{+1}, 3^{+2} *$
					$\begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & 8\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3$	19	$4_1^{+1}, 3^{+2} *$
					$(4\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 14\mathbb{A}_1$	19	$2_2^{-2}, 4_1^{+1}, 3^{+1} *$
					$(4\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 4\mathbb{A}_2 \amalg 6\mathbb{A}_1$	19	$4_1^{+1}, 3^{+2} *$
					$(4\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1} *$
					$(4\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$8_3^{-1}, 3^{+1} *$

<b>n</b>	<b> G </b>	<b>i</b>	<b>G</b>	<b>Deg</b>	<b>rk S</b>	<b>q<sub>S</sub></b>
18	12	4	$D_{12}$	$(\mathbb{A}_1, 2\mathbb{A}_1) \subset 3\mathbb{A}_1$	18	$2_7^{+3}, 4_7^{+1}, 3^{+4}$
				$(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$	18	$2_0^{+4}, 3^{-3} *$
				$(\mathbb{A}_1, 6\mathbb{A}_1) \subset 7\mathbb{A}_1$	18	$2_1^{-3}, 4_7^{+1}, 3^{+3} *$
				$(2\mathbb{A}_1, 3\mathbb{A}_1) \subset 5\mathbb{A}_1$	18	$2_1^{+3}, 4_7^{+1}, 3^{-3}$
				$((2\mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1)_I$	18	$4_4^{-2}, 3^{+3} *$
				$((2\mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1)_{II}$	18	$2_{II}^{-4}, 3^{+3}$
				$(2\mathbb{A}_1, 6\mathbb{A}_1) \subset 2\mathbb{D}_4$	18	$2_{II}^{-2}, 3^{+3} *$
				$(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 9\mathbb{A}_1$	18	$2_7^{+3}, 4_7^{+1}, 3^{-2} *$
				$(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_3$	18	$2_6^{-4}, 3^{+2} *$
				$(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$4_2^{+2}, 3^{+2} *$
				$(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 6\mathbb{A}_2$	18	$2_{II}^{-2}, 3^{-1}, 9^{-1} *$
				$(\mathbb{A}_1, 2\mathbb{A}_1, 3\mathbb{A}_1) \subset 6\mathbb{A}_1$	19	$2_0^{+2}, 4_7^{+1}, 3^{-3} *$
				$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 9\mathbb{A}_1$	19	$2_1^{+1}, 4_2^{+2}, 3^{+3}$
				$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{D}_4 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 2\mathbb{D}_4$	19	$2_7^{-3}, 3^{+3} *$
				$(\mathbb{A}_1, 3\mathbb{A}_1, 6\mathbb{A}_1) \subset 10\mathbb{A}_1$	19	$2_2^{-2}, 4_7^{+1}, 3^{-2} *$
				$\begin{pmatrix} \mathbb{A}_1 & 4\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 3\mathbb{A}_3$	19	$2_1^{+3}, 3^{+2} *$
				$(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 13\mathbb{A}_1$	19	$2_7^{+1}, 4_2^{+2}, 3^{+2} *$
				$(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$2_3^{+3}, 3^{-1}, 9^{-1} *$
				$\begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & 3\mathbb{A}_1 & 9\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 11\mathbb{A}_1$	19	$2_7^{+1}, 4_6^{+2}, 3^{-2}$
				$\begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 3\mathbb{A}_3$	19	$2_2^{+2}, 4_7^{+1}, 3^{+2} *$

				$\begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 3\mathbb{A}_3$	19	$2_{II}^{+2}, 4_1^{+1}, 3^{+2}$
				$\begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & 2\mathbb{D}_4 \\ & 3\mathbb{A}_1 & 9\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 2\mathbb{D}_4$	19	$2_5^{+3}, 3^{-2} *$
				$\begin{pmatrix} 2\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_{II} \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1$	19	$2_{II}^{+2}, 4_1^{+1}, 3^{+2}$
				$\begin{pmatrix} 2\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 6\mathbb{A}_1 & 6\mathbb{A}_2 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$4_3^{-1}, 3^{-1}, 9^{-1} *$
				$\begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{D}_4 & (8\mathbb{A}_1)_I \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{D}_4 \amalg 6\mathbb{A}_1$	19	$4_1^{+1}, 3^{+2} *$
				$(3\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 15\mathbb{A}_1$	19	$2_3^{-1}, 4_4^{-2}, 3^{-1} *$
				$(3\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$2_1^{-3}, 9^{-1} *$
				$\begin{pmatrix} 3\mathbb{A}_1 & 3\mathbb{A}_3 & 9\mathbb{A}_1 \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_3 \amalg 6\mathbb{A}_1$	19	$2_6^{-2}, 4_5^{-1}, 3^{+1} *$
				$\begin{pmatrix} 3\mathbb{A}_1 & 3\mathbb{A}_3 & 9\mathbb{A}_1 \\ & 6\mathbb{A}_1 & 6\mathbb{A}_2 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_5$	19	$2_3^{+3}, 3^{-1} *$
<b>n</b>	<b> G </b>	<b>i</b>	<b>G</b>	<b>Deg</b>	<b>rk S</b>	<b>q<sub>S</sub></b>
21	16	14	$C_2^4$	$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	17	$2_{II}^{+4}, 8_7^{+1} *$
				$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$2_{II}^{-2}, 4_2^{-2} *$
				$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1} *$

<b>n</b>	<b> G </b>	<b>i</b>	<b>G</b>	<b>Deg</b>	<b>rk S</b>	<b>q<sub>S</sub></b>
22	16	11	$C_2 \times D_8$	$(2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1$	18	$4_6^{+4} *$
				$(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$	18	$4_7^{+3}, 8_7^{+1} *$
				$(2\mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$	18	$4_6^{+4}$
				$((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_I$	18	$4_6^{+4} *$
				$((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_{II}$	18	$2_{II}^{-2}, 8_6^{-2} *$
				$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$2_{II}^{+2}, 4_7^{+1}, 8_7^{+1} *$
				$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$	18	$2_{II}^{-2}, 4_2^{-2} *$
				$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$	18	$2_{II}^{+2}, 4_6^{+2} *$
				$(2\mathbb{A}_1, 2\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	19	$4_6^{+2}, 8_7^{+1} *$
				$(2\mathbb{A}_1, 2\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$	19	$4_5^{+3} *$
				$(2\mathbb{A}_1, (4\mathbb{A}_1, 4\mathbb{A}_1)_{II}) \subset 10\mathbb{A}_1$	19	$4_5^{-1}, 8_4^{-2} *$
				$(2\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 14\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1}$
				$\begin{pmatrix} 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$4_5^{+3} *$
				$((4\mathbb{A}_1, 4\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$4_5^{+3} *$
				$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_{II}^{-2}, 8_5^{-1} *$
30	18	4	$\mathfrak{A}_{3,3}$	$(3\mathbb{A}_1, 3\mathbb{A}_1) \subset 6\mathbb{A}_1$	18	$2_2^{+2}, 3^{+2}, 9^{-1} *$
				$(3\mathbb{A}_1, 9\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$2_0^{+3}, 3^{-3} *$
				$(3\mathbb{A}_1, 9\mathbb{A}_1) \subset 3\mathbb{D}_4$	18	$3^{-3} *$
				$(9\mathbb{A}_1, 9\mathbb{A}_1) \subset 9\mathbb{A}_2$	18	$3^{-3} *$
				$(3\mathbb{A}_1, 3\mathbb{A}_1, 3\mathbb{A}_1) \subset 9\mathbb{A}_1$	19	$2_7^{-3}, 3^{-1}, 9^{-1} *$
				$(3\mathbb{A}_1, 3\mathbb{A}_1, 9\mathbb{A}_1) \subset 15\mathbb{A}_1$	19	$2_1^{+3}, 3^{+2} *$
				$\begin{pmatrix} 3\mathbb{A}_1 & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & 3\mathbb{A}_1 & 3\mathbb{D}_4 \\ & & 9\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 3\mathbb{D}_4$	19	$2_1^{+1}, 3^{+2} *$

<b>n</b>	<b> G </b>	<b>i</b>	<b>G</b>	<b>rk <math>S_G</math></b>	<b>Deg</b>	<b>rk <math>S</math></b>	<b>qs</b>
34	24	12	$\mathfrak{S}_4$	17	$(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$	19	$4_1^{+3}, 3^{+2} *$
					$(\mathbb{A}_1, 2\mathbb{A}_1) \subset 3\mathbb{A}_1$	19	$2_1^{+1}, 4_{II}^{+2}, 3^{+2} *$
					$(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$	19	$4_3^{-3}, 3^{-1} *$
					$(\mathbb{A}_1, 4\mathbb{A}_1) \subset 5\mathbb{A}_1$	19	$2_1^{+1}, 4_1^{+1}, 8_7^{+1}, 3^{+2} *$
					$(\mathbb{A}_1, (6\mathbb{A}_1)_I) \subset 7\mathbb{A}_1$	19	$2_1^{+1}, 4_6^{-2}, 3^{+1} *$
					$(\mathbb{A}_1, 8\mathbb{A}_1) \subset 9\mathbb{A}_1$	19	$2_1^{+1}, 4_0^{+2}, 3^{+2} *$
					$(\mathbb{A}_1, 12\mathbb{A}_1) \subset 13\mathbb{A}_1$	19	$2_7^{+1}, 4_5^{-1}, 8_7^{+1}, 3^{-1} *$
					$(\mathbb{A}_1, 6\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$2_7^{+1}, 4_{II}^{+2}, 3^{+1} *$
					$(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$	19	$2_2^{+2}, 8_7^{+1}, 3^{+2} *$
					$(2\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 8\mathbb{A}_1$	19	$4_7^{-3}, 3^{+1} *$
					$(2\mathbb{A}_1, 12\mathbb{A}_1) \subset 14\mathbb{A}_1$	19	$2_2^{+2}, 8_1^{+1}, 3^{-1} *$
					$(3\mathbb{A}_1, 4\mathbb{A}_1) \subset 7\mathbb{A}_1$	19	$2_1^{+1}, 4_1^{+1}, 8_5^{-1}, 3^{-1} *$
					$(3\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 3\mathbb{A}_3$	19	$4_5^{+3} *$
					$(3\mathbb{A}_1, 8\mathbb{A}_1) \subset 11\mathbb{A}_1$	19	$2_1^{+1}, 4_2^{-2}, 3^{-1} *$
					$(3\mathbb{A}_1, 12\mathbb{A}_1) \subset 15\mathbb{A}_1$	19	$2_7^{+1}, 4_7^{+1}, 8_7^{+1} *$
					$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	19	$2_{II}^{+2}, 4_1^{+1}, 3^{+2} *$
					$(4\mathbb{A}_1, (6\mathbb{A}_1)_I) \subset 10\mathbb{A}_1$	19	$2_6^{+2}, 8_1^{+1}, 3^{+1} *$
					$(4\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 10\mathbb{A}_1$	19	$2_{II}^{+2}, 8_3^{-1}, 3^{+1} *$
					$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$	19	$8_1^{+1}, 3^{+2} *$
					$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$	19	$4_1^{+1}, 3^{+2} *$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$2_{II}^{+2}, 4_3^{-1}, 3^{-1} *$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 4\mathbb{D}_4$	19	$4_3^{-1}, 3^{-1} *$
					$(4\mathbb{A}_1, 6\mathbb{A}_2) \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$8_3^{-1}, 3^{+1} *$
					$((6\mathbb{A}_1)_I, (6\mathbb{A}_1)_{II}) \subset 12\mathbb{A}_1$	19	$4_5^{+3} *$
					$((6\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 14\mathbb{A}_1$	19	$2_4^{-2}, 4_7^{+1}, 3^{+1} *$
					$((6\mathbb{A}_1)_I, 12\mathbb{A}_1) \subset 6\mathbb{A}_3$	19	$2_6^{-2}, 4_3^{-1} *$
39	32	27	$2^4 C_2$	17	$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1} *$
					$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$	19	$4_5^{+3} *$
					$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1} *$
40	32	49	$Q_8 * Q_8$	17	$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$4_5^{+3} *$
49	48	50	$2^4 C_3$	17	$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	19	$2_{II}^{-2}, 4_7^{+1}, 3^{-1} *$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1} *$

Table 3: Markings  $S \subset N_j$  by Niemeier lattices, and lattices  $S_{N_j}^\perp$  of degenerations of codimension  $\geq 2$  of Kählerian K3 surfaces with finite symplectic automorphism groups  $G = Clos(G)$  in notations [18], [19], [20] for  $\mathbf{n} \geq 12$ .

$\mathbf{n}=12$ , degeneration  $(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{12,1}$
orbits of	$(\alpha_1, \alpha_4)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=12$ , degeneration  $(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 8\mathbb{A}_2$ :

$j$	18	22 *
$H$	$H_{12,1}$	$H_{12,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,1})$	$(\alpha_{1,1}, \alpha_{2,1})$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$3A_1 \oplus A_2$

$\mathbf{n}=12$ , degeneration  $(8\mathbb{A}_1, \mathbb{A}_2) \subset 8\mathbb{A}_1 \amalg \mathbb{A}_2$ :

$j$	22 *
$H$	$H_{12,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,5}), (\alpha_{1,1}, \alpha_{1,8}), (\alpha_{1,1}, \alpha_{1,11}), (\alpha_{2,1}, \alpha_{1,5}), (\alpha_{2,1}, \alpha_{1,8}), (\alpha_{2,1}, \alpha_{1,11})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$

$\mathbf{n}=12$ , degeneration  $(\mathbb{A}_2, \mathbb{A}_2) \subset 2\mathbb{A}_2$ :

$j$	22 *
$H$	$H_{12,1}$
orbits of	$(\alpha_{1,5}, \alpha_{1,8}), (\alpha_{1,5}, \alpha_{1,11}), (\alpha_{1,8}, \alpha_{1,11})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_2$

$\mathbf{n}=16$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$ :

$j$	19	20	20	22	23
$H$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}), \dots$	$(\alpha_{1,1}, \alpha_{3,1}), \dots$	$(\alpha_{1,1}, \alpha_{4,1})$	$(\alpha_{1,1}, \alpha_{1,2}), \dots$	$(\alpha_1, \alpha_6), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$\{0\}$	$\{0\}$	$\{0\}$	$2A_1$

$\mathbf{n}=16$ , degeneration  $(\mathbb{A}_1, 5\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	19	19	19	19	19
$H$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}), \dots$	$(\alpha_{3,3}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{1,1}), \dots$	$(\alpha_{1,3}, \alpha_{2,1}), \dots$	$(\alpha_{2,3}, \alpha_{2,1})$
$(S_{N_j}^\perp)^{(2)}$	$3A_1$	$3A_1$	$3A_1$	$3A_1$	$3A_1$

20	20	20	20	22
$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
$(\alpha_{1,1}, \alpha_{1,2}), \dots$	$(\alpha_{2,1}, \alpha_{1,2}), \dots$	$(\alpha_{1,1}, \alpha_{2,2}), \dots$	$(\alpha_{2,1}, \alpha_{2,2}), \dots$	$(\alpha_{1,2}, \alpha_{1,3}), \dots$
$A_2$	$A_2$	$A_2$	$A_2$	$A_2$

22	22	23	23
$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
$(\alpha_{2,2}, \alpha_{1,3}), \dots$	$(\alpha_{1,1}, \alpha_{1,3}), \dots$	$(\alpha_1, \alpha_2), \dots$	$(\alpha_{19}, \alpha_2), \dots$
$A_2$	$A_2$	$3A_1$	$3A_1$

$\mathbf{n}=16$ , degeneration  $(5\mathbb{A}_1, 5\mathbb{A}_1) \subset 10\mathbb{A}_1$ :

$j$	19	20	20	22	23 *
$H$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1})$	$(\alpha_{1,2}, \alpha_{3,2}), \dots$	$(\alpha_{1,2}, \alpha_{4,2})$	$(\alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_2, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$A_4$	$A_4$	$2A_2$	$4A_1$

$\mathbf{n}=16$ , degeneration  $(5\mathbb{A}_1, 5\mathbb{A}_1) \subset 5\mathbb{A}_2$ :

$j$	19	20	20	22 *
$H$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{1,2}, \alpha_{2,2}), \dots$	$(\alpha_{2,2}, \alpha_{3,2})$	$(\alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$A_4$	$A_4$	$2A_2$

$\mathbf{n}=16$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, 5\mathbb{A}_1) \subset 7\mathbb{A}_1$ :

$j$	19	19	19
$H$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{1,1}), \dots$	$(\alpha_{3,3}, \alpha_{4,3}, \alpha_{1,1}), \dots$	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{2,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$

20	20	20	20
$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{1,2}), \dots$	$(\alpha_{1,1}, \alpha_{4,1}, \alpha_{1,2}), \dots$	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{2,2}), \dots$	$(\alpha_{1,1}, \alpha_{4,1}, \alpha_{2,2}), \dots$
$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

22	22	23	23
$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3}), \dots$	$(\alpha_{1,1}, \alpha_{2,2}, \alpha_{1,3}), \dots$	$(\alpha_1, \alpha_6, \alpha_2), \dots$	$(\alpha_1, \alpha_{19}, \alpha_2), \dots$
$\{0\}$	$\{0\}$	$2A_1$	$2A_1$

$\mathbf{n}=16$ , degeneration  $(\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 11\mathbb{A}_1$ :

$j$	19	19	19
$H$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}, \alpha_{3,1}), \dots$	$(\alpha_{4,3}, \alpha_{1,1}, \alpha_{3,1}), \dots$	$(\alpha_{2,3}, \alpha_{1,1}, \alpha_{3,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$3A_1$	$3A_1$	$3A_1$

20	20	20	20
$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{3,2}), \dots$	$(\alpha_{2,1}, \alpha_{1,2}, \alpha_{3,2}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{4,2}), \dots$	$(\alpha_{2,1}, \alpha_{1,2}, \alpha_{4,2}), \dots$
$A_2$	$A_2$	$A_2$	$A_2$

22	22	23	23
$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_{2,2}, \alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_1, \alpha_2, \alpha_3), \dots$	$(\alpha_6, \alpha_2, \alpha_3), \dots$
$A_2$	$A_2$	$3A_1$	$3A_1$

$\mathbf{n}=16$ , degeneration  $(\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 5\mathbb{A}_2$ :

$j$	19	19	19
$H$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{2,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{3,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$3A_1$	$3A_1$	$3A_1$

20	20	20	20
$H_{16,1}$	$H_{16,1}$	$H_{16,1}$	$H_{16,1}$
$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,2}), \dots$	$(\alpha_{2,1}, \alpha_{1,2}, \alpha_{2,2}), \dots$	$(\alpha_{1,1}, \alpha_{2,2}, \alpha_{3,2}), \dots$	$(\alpha_{2,1}, \alpha_{2,2}, \alpha_{3,2}), \dots$
$A_2$	$A_2$	$A_2$	$A_2$



22	22
$H_{16,1}$	$H_{16,1}$
$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{2,3}), \dots$	$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$A_2$	$A_2$

$\mathbf{n}=16$ , degeneration  $(5\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 15\mathbb{A}_1$ :

$j$	19	23 *
$H$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{4,1}), \dots$	$(\alpha_2, \alpha_3, \alpha_4), \dots$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$4A_1$

$\mathbf{n}=16$ , degeneration  $(5\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 5\mathbb{A}_2 \amalg 5\mathbb{A}_1$ :

$j$	20	22 *
$H$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,2}, \alpha_{2,2}, \alpha_{4,2}), \dots$	$(\alpha_{1,3}, \alpha_{2,3}, \alpha_{1,5}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_4$	$2A_2$

$\mathbf{n}=16$ , degeneration  $\begin{pmatrix} 5\mathbb{A}_1 & 5\mathbb{A}_2 & 10\mathbb{A}_1 \\ & 5\mathbb{A}_1 & 5\mathbb{A}_2 \\ & & 5\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_3$ :

$j$	19	20 *
$H$	$H_{16,1}$	$H_{16,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,1}, \alpha_{3,1}), \dots$	$(\alpha_{1,2}, \alpha_{2,2}, \alpha_{3,2}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$A_4$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$ :

$j$	19	21	21	23 *	23
$H$	$H_{17,2}$	$H_{17,1}$	$H_{17,2}$	$H_{17,1}$	$H_{17,2}$
orbits of	$(\alpha_{1,4}, \alpha_{3,4}), \dots$	$(\alpha_{1,1}, \alpha_{3,1}),$ $(\alpha_{1,5}, \alpha_{3,5})$	$(\alpha_{1,1}, \alpha_{3,1})$	$(\alpha_5, \alpha_{17}), \dots$	$(\alpha_5, \alpha_{17}),$ $(\alpha_6, \alpha_{10})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$A_3$	$\{0\}$	$3A_1$	$2A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$ :

$j$	23 *	23
$H$	$H_{17,1}$	$H_{17,3}$
orbits of	$(\alpha_{15}, \alpha_1)$	$(\alpha_{15}, \alpha_1), (\alpha_{10}, \alpha_6)$
$(S_{N_i}^\perp)^{(2)}$	$4A_1$	$A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1) \subset 5\mathbb{A}_1$ :

$j$	19	19	19	19	19
$H$	$H_{17,1}$	$H_{17,1}$	$H_{17,2}$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{2,3}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{2,1}),$ $(\alpha_{2,4}, \alpha_{2,1})$	$(\alpha_{2,3}, \alpha_{2,1})$	$(\alpha_{1,4}, \alpha_{2,1}), \dots$	$(\alpha_{2,4}, \alpha_{2,1})$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_2$	$D_4$	$3A_1 \oplus A_2$	$3A_1 \oplus A_2$

21	21	21	21	22	22
$H_{17,2}$	$H_{17,2}$	$H_{17,1}$	$H_{17,2}$	$H_{17,1}$	$H_{17,1}$
$(\alpha_{1,1}, \alpha_{1,3}), \dots$	$(\alpha_{2,1}, \alpha_{1,3}),$ $(\alpha_{2,1}, \alpha_{3,3})$	$(\alpha_{1,1}, \alpha_{2,3}),$ $(\alpha_{3,1}, \alpha_{2,3})$	$(\alpha_{2,1}, \alpha_{2,3})$	$(\alpha_{1,1}, \alpha_{1,3}), \dots$	$(\alpha_{2,1}, \alpha_{1,3}), \dots$
$A_1$	$A_1$	$A_1$	$A_1$	$A_2$	$A_2$

23	23	23 *	23
$H_{17,1}$	$H_{17,1}$	$H_{17,2}$	$H_{17,3}$
$(\alpha_5, \alpha_2), \dots$	$(\alpha_{15}, \alpha_2)$	$(\alpha_5, \alpha_2), \dots$	$(\alpha_{10}, \alpha_2), \dots$
$4A_1$	$4A_1$	$3A_1$	$A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 6\mathbb{A}_1) \subset 7\mathbb{A}_1$ :

$j$	21	21	21	21	22 *
$H$	$H_{17,1}$	$H_{17,1}$	$H_{17,2}$	$H_{17,2}$	$H_{17,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,2}), \dots$	$(\alpha_{2,1}, \alpha_{2,2}),$ $(\alpha_{2,5}, \alpha_{2,2})$	$(\alpha_{1,1}, \alpha_{1,2}),$ $(\alpha_{3,1}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{1,2})$	$(\alpha_{1,1}, \alpha_{1,5}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$A_1$	$A_1$	$A_2$

23	23
$H_{17,2}$	$H_{17,2}$
$(\alpha_5, \alpha_1), \dots$	$(\alpha_6, \alpha_1), \dots$
$3A_1$	$3A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 12\mathbb{A}_1) \subset 13\mathbb{A}_1$ :

$j$	19 *	19	19
$H$	$H_{17,2}$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{2,3}, \alpha_{1,1})$	$(\alpha_{1,4}, \alpha_{1,1}), \dots$	$(\alpha_{2,4}, \alpha_{1,1})$
$(S_{N_i}^\perp)^{(2)}$	$D_4$	$3A_1 \oplus A_2$	$3A_1 \oplus A_2$

21	21	23	23
$H_{17,1}$	$H_{17,1}$	$H_{17,1}$	$H_{17,1}$
$(\alpha_{1,1}, \alpha_{1,2}), \dots$	$(\alpha_{2,1}, \alpha_{1,2}), (\alpha_{2,5}, \alpha_{1,2})$	$(\alpha_5, \alpha_3), \dots$	$(\alpha_{15}, \alpha_3)$
$A_1 \oplus A_3$	$A_1 \oplus A_3$	$4A_1$	$4A_1$

$\mathbf{n}=17$ , degeneration  $(3\mathbb{A}_1, 4\mathbb{A}_1) \subset 7\mathbb{A}_1$ :

$j$	19	19	19
$H$	$H_{17,1}$	$H_{17,1}$	$H_{17,2}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}), \dots$	$(\alpha_{1,3}, \alpha_{2,1}), (\alpha_{1,4}, \alpha_{2,1})$	$(\alpha_{1,3}, \alpha_{2,1})$
$(S_{N_i}^\perp)^{(2)}$	$A_1 \oplus A_2$	$A_1 \oplus A_2$	$A_1 \oplus D_4$

21	21	23 *	23
$H_{17,2}$	$H_{17,2}$	$H_{17,1}$	$H_{17,3}$
$(\alpha_{2,2}, \alpha_{1,3}), (\alpha_{2,2}, \alpha_{3,3})$	$(\alpha_{2,2}, \alpha_{2,3})$	$(\alpha_1, \alpha_2)$	$(\alpha_1, \alpha_2), \dots$
$A_3$	$A_3$	$5A_1$	$2A_1$

$\mathbf{n}=17$ , degeneration  $(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{17,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2})$
$(S_{N_i}^\perp)^{(2)}$	$A_3$

$\mathbf{n}=17$ , degeneration  $(3\mathbb{A}_1, 12\mathbb{A}_1) \subset 15\mathbb{A}_1$ :

$j$	19	23 *
$H$	$H_{17,2}$	$H_{17,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1})$	$(\alpha_1, \alpha_3)$
$(S_{N_i}^\perp)^{(2)}$	$A_1 \oplus D_4$	$5A_1$

$\mathbf{n}=17$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	19	21	23 *	23
$H$	$H_{17,1}$	$H_{17,2}$	$H_{17,2}$	$H_{17,3}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1}), \dots$	$(\alpha_{1,3}, \alpha_{3,3})$	$(\alpha_2, \alpha_4)$	$(\alpha_2, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$A_3$	$4A_1$	$2A_1$

$\mathbf{n}=17$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 4\mathbb{A}_2$ :

$j$	19	21 *	22
$H$	$H_{17,1}$	$H_{17,2}$	$H_{17,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{1,3}, \alpha_{2,3}), (\alpha_{2,3}, \alpha_{3,3})$	$(\alpha_{1,3}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$A_3$	$2A_2$

$\mathbf{n}=17$ , degeneration  $(4\mathbb{A}_1, 6\mathbb{A}_1) \subset 10\mathbb{A}_1$ :

$j$	21	21	22	23 *
$H$	$H_{17,2}$	$H_{17,2}$	$H_{17,1}$	$H_{17,2}$
orbits of	$(\alpha_{1,3}, \alpha_{1,2}), (\alpha_{3,3}, \alpha_{1,2})$	$(\alpha_{2,3}, \alpha_{1,2})$	$(\alpha_{1,3}, \alpha_{1,5})$	$(\alpha_2, \alpha_1), \dots$
$(S_{N_i}^\perp)^{(2)}$	$A_3$	$A_3$	$2A_2$	$4A_1$

$\mathbf{n}=17$ , degeneration  $(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{17,1}$
orbits of	$(\alpha_2, \alpha_3)$
$(S_{N_i}^\perp)^{(2)}$	$5A_1$

$\mathbf{n}=17$ , degeneration  $(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 4\mathbb{D}_4$ :

$j$	19 *
$H$	$H_{17,2}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1})$
$(S_{N_i}^\perp)^{(2)}$	$A_2 \oplus D_4$

$\mathbf{n}=17$ , degeneration  $(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{17,2}$
orbits of	$(\alpha_1, \alpha_7)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=17$ , degeneration  $(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 6\mathbb{A}_2$ :

$j$	22 *
$H$	$H_{17,1}$
orbits of	$(\alpha_{1,5}, \alpha_{2,5})$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$

$\mathbf{n}=17$ , degeneration  $(6\mathbb{A}_1, 12\mathbb{A}_1) \subset 6\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{17,1}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2})$
$(S_{N_j}^\perp)^{(2)}$	$2A_3$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, \mathbb{A}_1) \subset 3\mathbb{A}_1$ :

$j$	19	23 *
$H$	$H_{17,2}$	$H_{17,1}$
orbits of	$(\alpha_{1,4}, \alpha_{3,4}, \alpha_{4,4})$	$(\alpha_5, \alpha_{17}, \alpha_{21}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_2$	$2A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	19	21	21
$H$	$H_{17,2}$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{1,4}, \alpha_{3,4}, \alpha_{2,1}), \dots$	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{1,3}), (\alpha_{1,1}, \alpha_{3,1}, \alpha_{3,3})$	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$\{0\}$	$\{0\}$

23 *	23
$H_{17,1}$	$H_{17,2}$
$(\alpha_5, \alpha_{17}, \alpha_2), \dots$	$(\alpha_5, \alpha_{17}, \alpha_2), \dots$
$3A_1$	$2A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	21	23 *
$H$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{1,2})$	$(\alpha_6, \alpha_{10}, \alpha_1), (\alpha_5, \alpha_{17}, \alpha_7)$
$(S_{N_j}^\perp)^{(2)}$	$\{0\}$	$2A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, 12\mathbb{A}_1) \subset 14\mathbb{A}_1$ :

$j$	19	21	23 *
$H$	$H_{17,2}$	$H_{17,1}$	$H_{17,1}$
orbits of	$(\alpha_{1,4}, \alpha_{3,4}, \alpha_{1,1}), \dots$	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{1,2}), (\alpha_{1,5}, \alpha_{3,5}, \alpha_{1,2})$	$(\alpha_5, \alpha_{17}, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$A_3$	$3A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 3\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	23 *	23
$H$	$H_{17,1}$	$H_{17,3}$
orbits of	$(\alpha_{15}, \alpha_1, \alpha_2)$	$(\alpha_{15}, \alpha_1, \alpha_2)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 3\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{17,1}$
orbits of	$(\alpha_{15}, \alpha_1, \alpha_3)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 9\mathbb{A}_1$ :

$j$	19	21	21
$H$	$H_{17,1}$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{2,3}, \alpha_{1,1}, \alpha_{3,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{3,3}), (\alpha_{3,1}, \alpha_{1,3}, \alpha_{3,3})$	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{3,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_1$	$A_1$

23 *	23
$H_{17,2}$	$H_{17,3}$
$(\alpha_5, \alpha_2, \alpha_4), \dots$	$(\alpha_{10}, \alpha_2, \alpha_3), \dots$
$3A_1$	$A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{A}_2$ :

$j$	19	21	21	22 *
$H$	$H_{17,1}$	$H_{17,2}$	$H_{17,2}$	$H_{17,1}$
orbits of	$(\alpha_{2,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{2,3}), \dots$	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{2,3}),$ $(\alpha_{2,1}, \alpha_{2,3}, \alpha_{3,3})$	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_1$	$A_1$	$A_2$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 11\mathbb{A}_1$ :

$j$	21	21	21	21
$H$	$H_{17,2}$	$H_{17,2}$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{1,2}), \dots$	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{1,2}),$ $(\alpha_{2,1}, \alpha_{3,3}, \alpha_{1,2})$	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{1,2}),$ $(\alpha_{3,1}, \alpha_{2,3}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{2,3}, \alpha_{1,2})$
$(S_{N_j}^\perp)^{(2)}$	$A_1$	$A_1$	$A_1$	$A_1$

22	22	23	23
$H_{17,1}$	$H_{17,1}$	$H_{17,2}$	$H_{17,2}$
$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_5, \alpha_2, \alpha_1), \dots$	$(\alpha_6, \alpha_2, \alpha_1), \dots$
$A_2$	$A_2$	$3A_1$	$3A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1, 12\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{D}_4$ :

$j$	19 *	19	19
$H$	$H_{17,2}$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{2,3}, \alpha_{2,1}, \alpha_{1,1})$	$(\alpha_{1,4}, \alpha_{1,2}, \alpha_{1,1}), \dots$	$(\alpha_{2,4}, \alpha_{1,2}, \alpha_{1,1})$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$A_2 \oplus 3A_1$	$A_2 \oplus 3A_1$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$ :

$j$	22 *
$H$	$H_{17,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,5}, \alpha_{2,5})$
$(S_{N_j}^\perp)^{(2)}$	$A_2$

$\mathbf{n}=17$ , degeneration  $(\mathbb{A}_1, 6\mathbb{A}_1, 12\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_3$ :

$j$	21	21
$H$	$H_{17,1}$	$H_{17,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,2}, \alpha_{1,2}), \dots$	$(\alpha_{2,1}, \alpha_{2,2}, \alpha_{1,2}), (\alpha_{2,5}, \alpha_{2,2}, \alpha_{1,2})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$

$\mathbf{n}=17$ , degeneration  $(3\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 11\mathbb{A}_1$ :

$j$	19	21	23 *
$H$	$H_{17,1}$	$H_{17,2}$	$H_{17,3}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}, \alpha_{3,1}), \dots$	$(\alpha_{2,2}, \alpha_{1,3}, \alpha_{3,3})$	$(\alpha_1, \alpha_2, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_2$	$A_3$	$2A_1$

$\mathbf{n}=17$ , degeneration  $(3\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_2$ :

$j$	19	21 *
$H$	$H_{17,1}$	$H_{17,2}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{2,2}, \alpha_{1,3}, \alpha_{2,3}), (\alpha_{2,2}, \alpha_{3,3}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_2$	$A_3$

$\mathbf{n}=17$ , degeneration  $(3\mathbb{A}_1, 4\mathbb{A}_1, 12\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 4\mathbb{D}_4$ :

$j$	19 *
$H$	$H_{17,2}$
orbits of	$(\alpha_{1,3}, \alpha_{2,1}, \alpha_{1,1})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus D_4$

$\mathbf{n}=17$ , degeneration  $(3\mathbb{A}_1, 6\mathbb{A}_1, 4\mathbb{A}_1) \subset 3\mathbb{A}_3 \amalg 4\mathbb{A}_1$ :

$j$	21	21
$H$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{1,3}), (\alpha_{2,2}, \alpha_{1,2}, \alpha_{3,3})$	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$

$\mathbf{n}=17$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$ :



$j$	19	23 *
$H$	$H_{17,1}$	$H_{17,3}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{4,1})$	$(\alpha_2, \alpha_3, \alpha_8)$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$2A_1$

$$\mathbf{n}=17, \text{ degeneration } \begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & 8\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3:$$

$j$	19	21 *
$H$	$H_{17,1}$	$H_{17,2}$
orbits of	$(\alpha_{1,1}, \alpha_{2,1}, \alpha_{3,1})$	$(\alpha_{1,3}, \alpha_{2,3}, \alpha_{3,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$A_3$

$$\mathbf{n}=17, \text{ degeneration } (4\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 14\mathbb{A}_1:$$

$j$	21	23 *
$H$	$H_{17,2}$	$H_{17,2}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{1,2})$	$(\alpha_2, \alpha_4, \alpha_1), (\alpha_2, \alpha_4, \alpha_7)$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$4A_1$

$$\mathbf{n}=17, \text{ degeneration } (4\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 4\mathbb{A}_2 \amalg 6\mathbb{A}_1:$$

$j$	21	22 *
$H$	$H_{17,2}$	$H_{17,1}$
orbits of	$(\alpha_{1,3}, \alpha_{2,3}, \alpha_{1,2}), (\alpha_{2,3}, \alpha_{3,3}, \alpha_{1,2})$	$(\alpha_{1,3}, \alpha_{2,3}, \alpha_{1,5}), (\alpha_{1,3}, \alpha_{2,3}, \alpha_{2,5})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$2A_2$

$$\mathbf{n}=17, \text{ degeneration } (4\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 16\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{17,2}$
orbits of	$(\alpha_2, \alpha_1, \alpha_7), (\alpha_4, \alpha_1, \alpha_7)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=17, \text{ degeneration } (4\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2:$$

$j$	22 *
$H$	$H_{17,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,5}, \alpha_{2,5}), (\alpha_{2,3}, \alpha_{1,5}, \alpha_{2,5})$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$

$\mathbf{n}=18$ , degeneration  $(\mathbb{A}_1, 2\mathbb{A}_1) \subset 3\mathbb{A}_1$ :

$j$	12	12	12	18	18	18
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1})$	$(\alpha_{4,1}, \alpha_{1,1})$	$(\alpha_{2,1}, \alpha_{3,1})$	$(\alpha_{3,1}, \alpha_{1,1})$	$(\alpha_{2,5}, \alpha_{1,1})$	$(\alpha_{2,5}, \alpha_{2,1})$
$(S_{N_j}^\perp)^{(2)}$	$A_1$	$A_1$	$A_1$	$A_2$	$A_1$	$A_1$

19	19	19	19	21
$H_{18,1}$	$H_{18,2}$	$H_{18,2}$	$H_{18,2}$	$H_{18,1}$
$(\alpha_{2,3}, \alpha_{2,1}), \dots$	$(\alpha_{4,3}, \alpha_{1,3})$	$(\alpha_{2,3}, \alpha_{2,4})$	$(\alpha_{4,3}, \alpha_{2,4})$	$(\alpha_{2,2}, \alpha_{1,1}), (\alpha_{2,1}, \alpha_{1,2})$
$A_2$	$A_1$	$A_1$	$A_1$	$A_1$

22	22	23
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{1,1}, \alpha_{1,7}), (\alpha_{2,1}, \alpha_{2,7})$	$(\alpha_{2,1}, \alpha_{1,7}), (\alpha_{1,1}, \alpha_{2,7})$	$(\alpha_1, \alpha_{15}), (\alpha_6, \alpha_{15}), (\alpha_1, \alpha_{16}), (\alpha_6, \alpha_{16})$
$\{0\}$	$\{0\}$	$A_1$

$\mathbf{n}=18$ , degeneration  $(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$ :

$j$	18	21 *
$H$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{3,1}, \alpha_{3,2})$	$(\alpha_{2,2}, \alpha_{2,3}), (\alpha_{2,1}, \alpha_{2,5})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$3A_1$

$\mathbf{n}=18$ , degeneration  $(\mathbb{A}_1, 6\mathbb{A}_1) \subset 7\mathbb{A}_1$ :

$j$	12	12	12	12	18	18
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{2,1}, \alpha_{1,2})$	$(\alpha_{4,1}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{3,2})$	$(\alpha_{4,1}, \alpha_{3,2})$	$(\alpha_{3,1}, \alpha_{1,2})$	$(\alpha_{2,5}, \alpha_{1,2})$
$(S_{N_j}^\perp)^{(2)}$	$3A_1$	$3A_1$	$3A_1$	$3A_1$	$2A_1 \oplus A_2$	$3A_1$

18	18	19 *	19	19	19
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,2}$	$H_{18,2}$	$H_{18,2}$
$(\alpha_{3,1}, \alpha_{2,2})$	$(\alpha_{2,5}, \alpha_{2,2})$	$(\alpha_{2,3}, \alpha_{1,1}), (\alpha_{2,4}, \alpha_{1,1})$	$(\alpha_{2,3}, \alpha_{1,1})$	$(\alpha_{4,3}, \alpha_{1,1})$	$(\alpha_{2,3}, \alpha_{1,4})$
$2A_1 \oplus A_2$	$3A_1$	$A_2$	$A_1$	$A_1$	$A_1$

19	21	21	22
$H_{18,2}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{4,3}, \alpha_{1,4})$	$(\alpha_{2,1}, \alpha_{1,3}), (\alpha_{2,2}, \alpha_{1,5})$	$(\alpha_{2,2}, \alpha_{1,3}), (\alpha_{2,1}, \alpha_{1,5})$	$(\alpha_{1,1}, \alpha_{1,5}), (\alpha_{2,1}, \alpha_{2,5})$
$A_1$	$3A_1$	$3A_1$	$\{0\}$

22	23
$H_{18,1}$	$H_{18,1}$
$(\alpha_{2,1}, \alpha_{1,5}), (\alpha_{1,1}, \alpha_{2,5})$	$(\alpha_1, \alpha_2), (\alpha_6, \alpha_2), (\alpha_1, \alpha_8), (\alpha_6, \alpha_8)$
$\{0\}$	$A_1$

$\mathbf{n}=18$ , degeneration  $(2\mathbb{A}_1, 3\mathbb{A}_1) \subset 5\mathbb{A}_1$ :

$j$	12	12	12	12	18	18
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,2})$	$(\alpha_{1,1}, \alpha_{4,2})$	$(\alpha_{3,1}, \alpha_{2,2})$	$(\alpha_{3,1}, \alpha_{4,2})$	$(\alpha_{1,1}, \alpha_{3,2})$	$(\alpha_{1,1}, \alpha_{1,5})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$	$A_3$	$A_3$	$A_1 \oplus A_2$	$2A_1$

18	18	19	19	19	19
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,2}$	$H_{18,2}$	$H_{18,2}$
$(\alpha_{2,1}, \alpha_{3,2})$	$(\alpha_{2,1}, \alpha_{1,5})$	$(\alpha_{2,1}, \alpha_{1,3}), \dots$	$(\alpha_{1,3}, \alpha_{2,1})$	$(\alpha_{2,4}, \alpha_{2,1})$	$(\alpha_{1,3}, \alpha_{3,1})$
$A_1 \oplus A_2$	$2A_1$	$A_1 \oplus A_2$	$2A_1$	$A_3$	$2A_1$

19	21	21	22
$H_{18,2}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{2,4}, \alpha_{3,1})$	$(\alpha_{1,1}, \alpha_{2,3}), (\alpha_{1,2}, \alpha_{2,5})$	$(\alpha_{1,1}, \alpha_{2,5}), (\alpha_{1,2}, \alpha_{2,3})$	$(\alpha_{1,7}, \alpha_{1,2}), (\alpha_{2,7}, \alpha_{2,2})$
$A_3$	$2A_1$	$2A_1$	$A_2$

22	23
$H_{18,1}$	$H_{18,1}$
$(\alpha_{2,7}, \alpha_{1,2}), (\alpha_{1,7}, \alpha_{2,2})$	$(\alpha_{15}, \alpha_4), (\alpha_{16}, \alpha_4), (\alpha_{15}, \alpha_5), (\alpha_{16}, \alpha_5)$
$A_2$	$2A_1$

$\mathbf{n}=18$ , degeneration  $((2\mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1)_I$ :

$j$	12	12	12	12	18	18
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2})$	$(\alpha_{1,1}, \alpha_{3,2})$	$(\alpha_{3,1}, \alpha_{1,2})$	$(\alpha_{3,1}, \alpha_{3,2})$	$(\alpha_{1,1}, \alpha_{1,2})$	$(\alpha_{1,1}, \alpha_{2,2})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$	$A_3$	$A_3$	$A_1 \oplus A_2$	$A_1 \oplus A_2$

18	18	19 *	19	19
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,2}$	$H_{18,2}$
$(\alpha_{2,1}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{2,2})$	$(\alpha_{2,5}, \alpha_{1,1}), (\alpha_{2,1}, \alpha_{1,5})$	$(\alpha_{2,4}, \alpha_{1,1})$	$(\alpha_{1,3}, \alpha_{1,4})$
$A_1 \oplus A_2$	$A_1 \oplus A_2$	$2A_2$	$A_3$	$2A_1$

21	22	22	23
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{1,1}, \alpha_{1,3}), (\alpha_{1,2}, \alpha_{1,5})$	$(\alpha_{1,7}, \alpha_{1,5}), (\alpha_{2,7}, \alpha_{2,5})$	$(\alpha_{2,7}, \alpha_{1,5}), (\alpha_{1,7}, \alpha_{2,5})$	$(\alpha_{16}, \alpha_2), (\alpha_{15}, \alpha_8)$
$2A_1$	$A_2$	$A_2$	$2A_1$

$\mathbf{n}=18$ , degeneration  $((2\mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1)_{II}$ :

$j$	19	21	23
$H$	$H_{18,2}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1})$	$(\alpha_{1,1}, \alpha_{1,5}), (\alpha_{1,2}, \alpha_{1,3})$	$(\alpha_{15}, \alpha_2), (\alpha_{16}, \alpha_8)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$

$\mathbf{n}=18$ , degeneration  $(2\mathbb{A}_1, 6\mathbb{A}_1) \subset 2\mathbb{D}_4$ :

$j$	19 *	19
$H$	$H_{18,1}$	$H_{18,2}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1}), (\alpha_{2,5}, \alpha_{1,5})$	$(\alpha_{2,4}, \alpha_{1,4})$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$A_3$

$\mathbf{n}=18$ , degeneration  $(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 9\mathbb{A}_1$ :

$j$	12	12	12	18	18	18
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2})$	$(\alpha_{4,2}, \alpha_{1,2})$	$(\alpha_{2,2}, \alpha_{3,2})$	$(\alpha_{3,2}, \alpha_{1,2})$	$(\alpha_{1,5}, \alpha_{1,2})$	$(\alpha_{1,5}, \alpha_{2,2})$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$D_4$	$D_4$	$3A_1 \oplus A_2$	$4A_1$	$4A_1$

19	19	19	19	21
$H_{18,1}$	$H_{18,2}$	$H_{18,2}$	$H_{18,2}$	$H_{18,1}$
$(\alpha_{1,3}, \alpha_{1,1}) \dots$	$(\alpha_{3,1}, \alpha_{1,1})$	$(\alpha_{2,1}, \alpha_{1,4})$	$(\alpha_{3,1}, \alpha_{1,4})$	$(\alpha_{2,5}, \alpha_{1,3}), (\alpha_{2,3}, \alpha_{1,5})$
$A_1 \oplus A_2$	$A_3$	$A_3$	$A_3$	$4A_1$

22	22	23 *
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{1,2}, \alpha_{1,5}), (\alpha_{2,2}, \alpha_{2,5})$	$(\alpha_{1,2}, \alpha_{2,5}), (\alpha_{2,2}, \alpha_{1,5})$	$(\alpha_2, \alpha_4), (\alpha_2, \alpha_5), (\alpha_8, \alpha_4), (\alpha_8, \alpha_5)$
$A_2$	$A_2$	$2A_1$

$\mathbf{n}=18$ , degeneration  $(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_3$ :

12	18	19	21 *
$H_{18,1}$	$H_{18,1}$	$H_{18,2}$	$H_{18,1}$
$(\alpha_{4,2}, \alpha_{3,2})$	$(\alpha_{3,2}, \alpha_{2,2})$	$(\alpha_{2,1}, \alpha_{1,1})$	$(\alpha_{2,3}, \alpha_{1,3}), (\alpha_{2,5}, \alpha_{1,5})$
$D_4$	$3A_1 \oplus A_2$	$A_3$	$4A_1$

$\mathbf{n}=18$ , degeneration  $(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	19	19	21	23 *
$H$	$H_{18,1}$	$H_{18,2}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,5})$	$(\alpha_{1,1}, \alpha_{1,4})$	$(\alpha_{1,3}, \alpha_{1,5})$	$(\alpha_2, \alpha_8)$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$A_3$	$4A_1$	$2A_1$

$\mathbf{n}=18$ , degeneration  $(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 6\mathbb{A}_2$ :

$j$	12	18	22 *
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,2}, \alpha_{3,2})$	$(\alpha_{1,2}, \alpha_{2,2})$	$(\alpha_{1,5}, \alpha_{2,5})$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$3A_1 \oplus A_2$	$A_2$

$\mathbf{n}=18$ , degeneration  $(\mathbb{A}_1, 2\mathbb{A}_1, 3\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	18 *	21	23
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{3,1}, \alpha_{1,1}, \alpha_{3,2})$	$(\alpha_{2,2}, \alpha_{1,1}, \alpha_{2,3}), (\alpha_{2,1}, \alpha_{1,2}, \alpha_{2,5})$	$(\alpha_1, \alpha_{15}, \alpha_4), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_1$	$A_1$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 9\mathbb{A}_1:$$

$j$	12	12	12	12
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1}, \alpha_{1,2})$	$(\alpha_{4,1}, \alpha_{1,1}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{1,1}, \alpha_{3,2})$	$(\alpha_{4,1}, \alpha_{1,1}, \alpha_{3,2})$
$(S_{N_j}^\perp)^{(2)}$	$A_1$	$A_1$	$A_1$	$A_1$

12	12	18	18	18
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{2,1}, \alpha_{3,1}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{3,1}, \alpha_{3,2})$	$(\alpha_{3,1}, \alpha_{1,1}, \alpha_{1,2})$	$(\alpha_{2,5}, \alpha_{1,1}, \alpha_{1,2})$	$(\alpha_{3,1}, \alpha_{1,1}, \alpha_{2,2})$
$A_1$	$A_1$	$A_2$	$A_1$	$A_2$

18	18	18	19	19
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,2}$
$(\alpha_{2,5}, \alpha_{1,1}, \alpha_{2,2})$	$(\alpha_{2,5}, \alpha_{2,1}, \alpha_{1,2})$	$(\alpha_{2,5}, \alpha_{2,1}, \alpha_{2,2})$	$(\alpha_{2,3}, \alpha_{2,5}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{2,4}, \alpha_{1,1})$
$A_1$	$A_1$	$A_1$	$A_2$	$A_1$

19	19	21	22	22
$H_{18,2}$	$H_{18,2}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{4,3}, \alpha_{2,4}, \alpha_{1,1})$	$(\alpha_{4,3}, \alpha_{1,3}, \alpha_{1,4})$	$(\alpha_{2,2}, \alpha_{1,1}, \alpha_{1,3}),$ $(\alpha_{2,1}, \alpha_{1,2}, \alpha_{1,5})$	$(\alpha_{1,1}, \alpha_{1,7}, \alpha_{1,5}),$ $(\alpha_{2,2}, \alpha_{2,7}, \alpha_{2,5})$	$(\alpha_{2,1}, \alpha_{1,7}, \alpha_{1,5}),$ $(\alpha_{1,1}, \alpha_{2,7}, \alpha_{2,5})$
$A_1$	$A_1$	$A_1$	$\{0\}$	$\{0\}$

22	22	23
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{1,1}, \alpha_{2,7}, \alpha_{1,5}), (\alpha_{2,1}, \alpha_{1,7}, \alpha_{2,5})$	$(\alpha_{2,1}, \alpha_{2,7}, \alpha_{1,5}), (\alpha_{1,1}, \alpha_{1,7}, \alpha_{2,5})$	$(\alpha_1, \alpha_{16}, \alpha_2), \dots$
$\{0\}$	$\{0\}$	$A_1$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{D}_4 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 2\mathbb{D}_4:$$

$j$	19 *	19	19
$H$	$H_{18,1}$	$H_{18,2}$	$H_{18,2}$
orbits of	$(\alpha_{2,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{2,4}, \alpha_{1,4})$	$(\alpha_{4,3}, \alpha_{2,4}, \alpha_{1,4}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_1$	$A_1$

$\mathbf{n}=18$ , degeneration  $(\mathbb{A}_1, 3\mathbb{A}_1, 6\mathbb{A}_1) \subset 10\mathbb{A}_1$ :

$j$	18	21	23 *
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{3,1}, \alpha_{3,2}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{2,5}, \alpha_{1,3}), (\alpha_{2,2}, \alpha_{2,3}, \alpha_{1,5})$	$(\alpha_1, \alpha_4, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$3A_1$	$A_1$

$\mathbf{n}=18$ , degeneration  $\begin{pmatrix} \mathbb{A}_1 & 4\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 3\mathbb{A}_3$ :

$j$	18	21 *
$H$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{3,1}, \alpha_{3,2}, \alpha_{2,2})$	$(\alpha_{2,2}, \alpha_{2,3}, \alpha_{1,3}), (\alpha_{2,1}, \alpha_{2,5}, \alpha_{1,5})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$3A_1$

$\mathbf{n}=18$ , degeneration  $(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 13\mathbb{A}_1$ :

$j$	19	19	19
$H$	$H_{18,1}$	$H_{18,2}$	$H_{18,2}$
orbits of	$(\alpha_{2,3}, \alpha_{1,1}, \alpha_{1,5}), (\alpha_{2,4}, \alpha_{1,1}, \alpha_{1,5})$	$(\alpha_{2,3}, \alpha_{1,1}, \alpha_{1,4})$	$(\alpha_{4,3}, \alpha_{1,1}, \alpha_{1,4})$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_1$	$A_1$

21 *	23
$H_{18,1}$	$H_{18,1}$
$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{1,5}), (\alpha_{2,2}, \alpha_{1,3}, \alpha_{1,5})$	$(\alpha_1, \alpha_2, \alpha_8), (\alpha_6, \alpha_2, \alpha_8)$
$3A_1$	$A_1$

$\mathbf{n}=18$ , degeneration  $(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$ :

$j$	12	12	18 *	18
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{2,1}, \alpha_{1,2}, \alpha_{3,2})$	$(\alpha_{4,1}, \alpha_{1,2}, \alpha_{3,2})$	$(\alpha_{3,1}, \alpha_{1,2}, \alpha_{2,2})$	$(\alpha_{2,5}, \alpha_{1,2}, \alpha_{2,2})$
$(S_{N_j}^\perp)^{(2)}$	$3A_1$	$3A_1$	$2A_1 \oplus A_2$	$3A_1$

22
$H_{18,1}$
$(\alpha_{1,1}, \alpha_{1,5}, \alpha_{2,5}), (\alpha_{2,1}, \alpha_{1,5}, \alpha_{2,5})$
$\{0\}$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & 3\mathbb{A}_1 & 9\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 11\mathbb{A}_1:$$

$j$	12	12	12	12
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,2}, \alpha_{1,2})$	$(\alpha_{1,1}, \alpha_{4,2}, \alpha_{1,2})$	$(\alpha_{1,1}, \alpha_{2,2}, \alpha_{3,2})$	$(\alpha_{3,1}, \alpha_{2,2}, \alpha_{1,2})$
$(S_{N_i}^\perp)^{(2)}$	$A_3$	$A_3$	$A_3$	$A_3$

12	12	18	18	18
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{3,1}, \alpha_{4,2}, \alpha_{1,2})$	$(\alpha_{3,1}, \alpha_{2,2}, \alpha_{3,2})$	$(\alpha_{1,1}, \alpha_{3,2}, \alpha_{1,2})$	$(\alpha_{1,1}, \alpha_{1,5}, \alpha_{1,2})$	$(\alpha_{1,1}, \alpha_{1,5}, \alpha_{2,2})$
$A_3$	$A_3$	$A_1 \oplus A_2$	$2A_1$	$2A_1$

18	18	18	19	19
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,2}$
$(\alpha_{2,1}, \alpha_{3,2}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{1,5}, \alpha_{1,2})$	$(\alpha_{2,1}, \alpha_{1,5}, \alpha_{2,2})$	$(\alpha_{2,5}, \alpha_{1,3}, \alpha_{1,1}), \dots$	$(\alpha_{2,4}, \alpha_{3,1}, \alpha_{1,1})$
$A_1 \oplus A_2$	$2A_1$	$2A_1$	$A_1 \oplus A_2$	$A_3$

19	19	21	22	22
$H_{18,2}$	$H_{18,2}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{1,3}, \alpha_{2,1}, \alpha_{1,4})$	$(\alpha_{1,3}, \alpha_{3,1}, \alpha_{1,4})$	$(\alpha_{1,1}, \alpha_{2,5}, \alpha_{1,3}),$ $(\alpha_{1,2}, \alpha_{2,3}, \alpha_{1,5})$	$(\alpha_{1,7}, \alpha_{1,2}, \alpha_{1,5}),$ $(\alpha_{2,7}, \alpha_{2,2}, \alpha_{2,5})$	$(\alpha_{2,7}, \alpha_{1,2}, \alpha_{1,5}),$ $(\alpha_{1,7}, \alpha_{2,2}, \alpha_{2,5})$
$2A_1$	$2A_1$	$2A_1$	$A_2$	$A_2$

22	22	23
$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
$(\alpha_{2,7}, \alpha_{1,2}, \alpha_{2,5}), (\alpha_{1,7}, \alpha_{2,2}, \alpha_{1,5})$	$(\alpha_{1,7}, \alpha_{1,2}, \alpha_{2,5}), (\alpha_{2,7}, \alpha_{2,2}, \alpha_{1,5})$	$(\alpha_{16}, \alpha_4, \alpha_2), \dots$
$A_2$	$A_2$	$2A_1$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 3\mathbb{A}_3:$$



$j$	12	12	18	18
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,1}, \alpha_{4,2}, \alpha_{3,2})$	$(\alpha_{3,1}, \alpha_{4,2}, \alpha_{3,2})$	$(\alpha_{1,1}, \alpha_{3,2}, \alpha_{2,2})$	$(\alpha_{2,1}, \alpha_{3,2}, \alpha_{2,2})$
$(S_{N_i}^\perp)^{(2)}$	$A_3$	$A_3$	$A_1 \oplus A_2$	$A_1 \oplus A_2$

19	21 *
$H_{18,2}$	$H_{18,1}$
$(\alpha_{2,4}, \alpha_{2,1}, \alpha_{1,1})$	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{1,3}), (\alpha_{1,2}, \alpha_{2,5}, \alpha_{1,5})$
$A_3$	$2A_1$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 3\mathbb{A}_3:$$

$j$	19	21
$H$	$H_{18,2}$	$H_{18,1}$
orbits of	$(\alpha_{1,3}, \alpha_{2,1}, \alpha_{1,1})$	$(\alpha_{1,1}, \alpha_{2,5}, \alpha_{1,5}), (\alpha_{1,2}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & 2\mathbb{D}_4 \\ & 3\mathbb{A}_1 & 9\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 2\mathbb{D}_4:$$

$j$	19 *	19	19
$H$	$H_{18,1}$	$H_{18,2}$	$H_{18,2}$
orbits of	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{1,1}), \dots$	$(\alpha_{2,4}, \alpha_{2,1}, \alpha_{1,4})$	$(\alpha_{2,4}, \alpha_{3,1}, \alpha_{1,4})$
$(S_{N_i}^\perp)^{(2)}$	$A_1 \oplus A_2$	$A_3$	$A_3$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_{II} \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1:$$

$j$	19	21	23
$H$	$H_{18,2}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,4}, \alpha_{1,1})$	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{1,5}),$ $(\alpha_{1,2}, \alpha_{1,3}, \alpha_{1,5})$	$(\alpha_{15}, \alpha_8, \alpha_2),$ $(\alpha_{16}, \alpha_2, \alpha_8)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 6\mathbb{A}_1 & 6\mathbb{A}_2 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 6\mathbb{A}_2:$$

$j$	12	12	18	18
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{3,2})$	$(\alpha_{3,1}, \alpha_{1,2}, \alpha_{3,2})$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,2})$	$(\alpha_{2,1}, \alpha_{1,2}, \alpha_{2,2})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$	$A_1 \oplus A_2$	$A_1 \oplus A_2$

22 *
$H_{18,1}$
$(\alpha_{1,7}, \alpha_{1,5}, \alpha_{2,5}), (\alpha_{2,7}, \alpha_{1,5}, \alpha_{2,5})$
$A_2$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{D}_4 & (8\mathbb{A}_1)_I \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{D}_4 \amalg 6\mathbb{A}_1:$$

$j$	19 *	19
$H$	$H_{18,1}$	$H_{18,2}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1}, \alpha_{1,5}), (\alpha_{2,5}, \alpha_{1,5}, \alpha_{1,1})$	$(\alpha_{2,4}, \alpha_{1,4}, \alpha_{1,1})$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$A_3$

$$\mathbf{n}=18, \text{ degeneration } (3\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 15\mathbb{A}_1:$$

$j$	19	19	23 *
$H$	$H_{18,1}$	$H_{18,2}$	$H_{18,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}, \alpha_{1,5}), (\alpha_{1,4}, \alpha_{1,1}, \alpha_{1,5})$	$(\alpha_{3,1}, \alpha_{1,1}, \alpha_{1,4})$	$(\alpha_4, \alpha_2, \alpha_8), (\alpha_5, \alpha_2, \alpha_8)$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_2$	$A_3$	$2A_1$

$$\mathbf{n}=18, \text{ degeneration } (3\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 6\mathbb{A}_2:$$

$j$	18	18 *	22
$H$	$H_{18,1}$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{1,5}, \alpha_{1,2}, \alpha_{2,2})$	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{3,2})$	$(\alpha_{1,2}, \alpha_{1,5}, \alpha_{2,5}), (\alpha_{2,2}, \alpha_{1,5}, \alpha_{2,5})$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$D_4$	$A_2$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 3\mathbb{A}_1 & 3\mathbb{A}_3 & 9\mathbb{A}_1 \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_3 \amalg 6\mathbb{A}_1:$$

$j$	19	21 *
$H$	$H_{18,2}$	$H_{18,1}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1}, \alpha_{1,4})$	$(\alpha_{2,3}, \alpha_{1,3}, \alpha_{1,5}), (\alpha_{2,5}, \alpha_{1,5}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$4A_1$

$$\mathbf{n}=18, \text{ degeneration } \begin{pmatrix} 3\mathbb{A}_1 & 3\mathbb{A}_3 & 9\mathbb{A}_1 \\ & 6\mathbb{A}_1 & 6\mathbb{A}_2 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_5:$$

$j$	12	18 *
$H$	$H_{18,1}$	$H_{18,1}$
orbits of	$(\alpha_{4,2}, \alpha_{3,2}, \alpha_{1,2})$	$(\alpha_{3,2}, \alpha_{2,2}, \alpha_{1,2})$
$(S_{N_j}^\perp)^{(2)}$	$D_4$	$3A_1 \oplus A_2$

$$\mathbf{n}=21, \text{ degeneration } (4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{21,1}$
orbits of	$(\alpha_1, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=21, \text{ degeneration } (4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{21,1}$
orbits of	$(\alpha_1, \alpha_2, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=21, \text{ degeneration } (4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{21,1}$
orbits of	$(\alpha_1, \alpha_2, \alpha_3, \alpha_8), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=22$ , degeneration  $(2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{22,2}$
orbits of	$(\alpha_{12}, \alpha_{16})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=22$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	21	21	23	23 *
$H$	$H_{22,1}$	$H_{22,1}$	$H_{22,2}$	$H_{22,3}$
orbits of	$(\alpha_{1,1}, \alpha_{2,3}),$ $(\alpha_{1,2}, \alpha_{2,3})$	$(\alpha_{1,1}, \alpha_{1,4})$ $(\alpha_{1,2}, \alpha_{1,4})$	$(\alpha_{12}, \alpha_6), \dots$	$(\alpha_8, \alpha_1), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$	$\{0\}$

$\mathbf{n}=22$ , degeneration  $(2\mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$ :

$j$	21	23
$H$	$H_{22,1}$	$H_{22,2}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}), (\alpha_{1,2}, \alpha_{1,3})$	$(\alpha_{12}, \alpha_2), (\alpha_{16}, \alpha_2)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$\mathbf{n}=22$ , degeneration  $((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_I$ :

$j$	23 *
$H$	$H_{22,2}$
orbits of	$(\alpha_6, \alpha_8)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=22$ , degeneration  $((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_{II}$ :

$j$	21	23 *
$H$	$H_{22,1}$	$H_{22,3}$
orbits of	$(\alpha_{2,3}, \alpha_{1,4})$	$(\alpha_1, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$\{0\}$

$\mathbf{n}=22$ , degeneration  $(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	21	23 *
$H$	$H_{22,1}$	$H_{22,2}$
orbits of	$(\alpha_{1,4}, \alpha_{1,3})$	$(\alpha_6, \alpha_2), (\alpha_8, \alpha_2)$
$(S_{N_i}^\perp)^{(2)}$	$4A_1$	$2A_1$

$\mathbf{n}=22$ , degeneration  $(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{22,1}$
orbits of	$(\alpha_{2,3}, \alpha_{1,3})$
$(S_{N_i}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=22$ , degeneration  $(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{22,1}$
orbist of	$(\alpha_2, \alpha_4)$
$(S_{N_i}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=22$ , degeneration  $(2\mathbb{A}_1, 2\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{22,2}$
orbits of	$(\alpha_{12}, \alpha_{16}, \alpha_6), (\alpha_{12}, \alpha_{16}, \alpha_8)$
$(S_{N_i}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=22$ , degeneration  $(2\mathbb{A}_1, 2\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{22,2}$
orbits of	$(\alpha_{12}, \alpha_{16}, \alpha_2)$
$(S_{N_i}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=22$ , degeneration  $(2\mathbb{A}_1, (4\mathbb{A}_1, 4\mathbb{A}_1)_{II}) \subset 10\mathbb{A}_1$ :

$j$	21 *	23
$H$	$H_{22,1}$	$H_{22,3}$
orbits of	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{1,4}), (\alpha_{1,2}, \alpha_{2,3}, \alpha_{1,4})$	$(\alpha_8, \alpha_1, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$\{0\}$

$\mathbf{n}=22$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 14\mathbb{A}_1$ :

$j$	21	23
$H$	$H_{22,1}$	$H_{22,2}$
orbits of	$(\alpha_{1,1}, \alpha_{1,4}, \alpha_{1,3}), \dots$	$(\alpha_{12}, \alpha_6, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$\mathbf{n}=22$ , degeneration  $\begin{pmatrix} 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3 :$

$j$	21 *
$H$	$H_{22,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{1,3}), (\alpha_{1,2}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=22$ , degeneration  $((4\mathbb{A}_1, 4\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{22,2}$
orbits of	$(\alpha_6, \alpha_8, \alpha_2)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=22$ , degeneration  $\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3 :$

$j$	21 *
$H$	$H_{22,1}$
orbits of	$(\alpha_{1,4}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=30$ , degeneration  $(3\mathbb{A}_1, 3\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	19	19	22	22	23 *
$H$	$H_{30,1}$	$H_{30,1}$	$H_{30,2}$	$H_{30,2}$	$H_{30,1}$
orbits of	$(\alpha_{2,1}, \alpha_{1,4}), \dots$	$(\alpha_{1,4}, \alpha_{1,5}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}), \dots$	$(\alpha_{1,1}, \alpha_{2,2}), \dots$	$(\alpha_4, \alpha_5), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus 2A_2$	$2A_1 \oplus A_2$	$\{0\}$	$\{0\}$	$3A_1$

$\mathbf{n}=30$ , degeneration  $(3\mathbb{A}_1, 9\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	19	23 *
$H$	$H_{30,1}$	$H_{30,1}$
orbits of	$(\alpha_{1,4}, \alpha_{1,1}), \dots$	$(\alpha_4, \alpha_1), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus 2A_2$	$3A_1$

$\mathbf{n}=30$ , degeneration  $(3\mathbb{A}_1, 9\mathbb{A}_1) \subset 3\mathbb{D}_4$ :

$j$	19 *
$H$	$H_{30,1}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1})$
$(S_{N_j}^\perp)^{(2)}$	$3A_2$

$\mathbf{n}=30$ , degeneration  $(9\mathbb{A}_1, 9\mathbb{A}_1) \subset 9\mathbb{A}_2$ :

$j$	22 *
$H$	$H_{30,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,1})$
$(S_{N_j}^\perp)^{(2)}$	$3A_2$

$\mathbf{n}=30$ , degeneration  $(3\mathbb{A}_1, 3\mathbb{A}_1, 3\mathbb{A}_1) \subset 9\mathbb{A}_1$ :

$j$	19 *	19	22
$H$	$H_{30,1}$	$H_{30,1}$	$H_{30,2}$
orbits of	$(\alpha_{2,1}, \alpha_{1,4}, \alpha_{1,5}), \dots$	$(\alpha_{1,4}, \alpha_{1,5}, \alpha_{1,6}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,6}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$3A_1$	$\{0\}$

22	22	23
$H_{30,2}$	$H_{30,2}$	$H_{30,1}$
$(\alpha_{1,1}, \alpha_{2,2}, \alpha_{2,6}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,6}), \dots$	$(\alpha_4, \alpha_5, \alpha_{11}), \dots$
$\{0\}$	$\{0\}$	$3A_1$

$\mathbf{n}=30$ , degeneration  $(3\mathbb{A}_1, 3\mathbb{A}_1, 9\mathbb{A}_1) \subset 15\mathbb{A}_1$ :

$j$	19	23 *
$H$	$H_{30,1}$	$H_{30,1}$
orbits of	$(\alpha_{1,4}, \alpha_{1,5}, \alpha_{1,1}), \dots$	$(\alpha_4, \alpha_5, \alpha_1), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$3A_1$

$\mathbf{n}=30$ , degeneration  $\begin{pmatrix} 3\mathbb{A}_1 & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & 3\mathbb{A}_1 & 3\mathbb{D}_4 \\ & & 9\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 3\mathbb{D}_4$  :

$j$	19 *
$H$	$H_{30,1}$
orbits of	$(\alpha_{1,4}, \alpha_{2,1}, \alpha_{1,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus 2A_2$

$\mathbf{n}=34$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$ :

$j$	21	21	23	23 *	23
$H$	$H_{34,1}$	$H_{34,3}$	$H_{34,1}$	$H_{34,2}$	$H_{34,3}$
orbits of	$(\alpha_{1,2}, \alpha_{3,2})$	$(\alpha_{1,2}, \alpha_{3,2})$	$(\alpha_5, \alpha_{17}), (\alpha_6, \alpha_{21})$	$(\alpha_{21}, \alpha_{24})$	$(\alpha_5, \alpha_{21})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$\{0\}$	$2A_1$	$A_1$	$\{0\}$

$\mathbf{n}=34$ , degeneration  $(\mathbb{A}_1, 2\mathbb{A}_1) \subset 3\mathbb{A}_1$ :

$j$	19	23 *
$H$	$H_{34,2}$	$H_{34,2}$
orbits of	$(\alpha_{1,4}, \alpha_{3,4})$	$(\alpha_{21}, \alpha_4), (\alpha_{24}, \alpha_4)$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_2$	$2A_1$

$\mathbf{n}=34$ , degeneration  $(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$ :



$j$	23 *	23
$H$	$H_{34,2}$	$H_{34,4}$
orbits of	$(\alpha_6, \alpha_2)$	$(\alpha_{10}, \alpha_2), (\alpha_{24}, \alpha_3)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$A_1$

$\mathbf{n}=34$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1) \subset 5\mathbb{A}_1$ :

$j$	19	19	19 *	19	19	21
$H$	$H_{34,1}$	$H_{34,1}$	$H_{34,2}$	$H_{34,2}$	$H_{34,2}$	$H_{34,2}$
orbits of	$(\alpha_{2,3}, \alpha_{2,1}),$ $(\alpha_{2,4}, \alpha_{2,1})$	$(\alpha_{2,3}, \alpha_{4,1}),$ $(\alpha_{2,4}, \alpha_{4,1})$	$(\alpha_{2,3}, \alpha_{2,1})$	$(\alpha_{1,4}, \alpha_{2,1})$	$(\alpha_{2,4}, \alpha_{2,1})$	$(\alpha_{2,1}, \alpha_{1,4}),$ $(\alpha_{2,1}, \alpha_{3,4})$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_2$	$A_3$	$A_1 \oplus A_2$	$A_1 \oplus A_2$	$A_1$

21	21	21	22	22	22	22
$H_{34,2}$ $(\alpha_{2,1}, \alpha_{2,4})$	$H_{34,3}$ $(\alpha_{1,2}, \alpha_{2,4}),$ $(\alpha_{3,2}, \alpha_{2,4})$	$H_{34,3}$ $(\alpha_{2,2}, \alpha_{2,4})$	$H_{34,1}$ $(\alpha_{1,6}, \alpha_{1,1}),$ $(\alpha_{2,6}, \alpha_{1,1})$	$H_{34,1}$ $(\alpha_{1,10}, \alpha_{1,1}),$ $(\alpha_{2,10}, \alpha_{1,1})$	$H_{34,1}$ $(\alpha_{1,6}, \alpha_{2,1}),$ $(\alpha_{2,6}, \alpha_{2,1})$	$H_{34,1}$ $(\alpha_{1,10}, \alpha_{2,1}),$ $(\alpha_{2,10}, \alpha_{2,1})$
$A_1$	$A_1$	$A_1$	$A_2$	$A_2$	$A_2$	$A_2$

23	23	23	23
$H_{34,2}$	$H_{34,2}$	$H_{34,3}$	$H_{34,4}$
$(\alpha_6, \alpha_1)$	$(\alpha_{21}, \alpha_1),$ $(\alpha_{24}, \alpha_1)$	$(\alpha_5, \alpha_1), (\alpha_5, \alpha_{10}),$ $(\alpha_{21}, \alpha_1), (\alpha_{21}, \alpha_{10})$	$(\alpha_{10}, \alpha_1), (\alpha_{10}, \alpha_5),$ $(\alpha_{24}, \alpha_1), (\alpha_{24}, \alpha_2)$
$2A_1$	$2A_1$	$A_1$	$A_1$

$\mathbf{n}=34$ , degeneration  $(\mathbb{A}_1, (6\mathbb{A}_1)_I) \subset 7\mathbb{A}_1$ :

$j$	21	23 *
$H$	$H_{34,1}$	$H_{34,1}$
orbits of	$(\alpha_{2,1}, \alpha_{2,3})$	$(\alpha_5, \alpha_1), (\alpha_{17}, \alpha_1), (\alpha_6, \alpha_8), (\alpha_{21}, \alpha_8)$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$3A_1$

$\mathbf{n}=34$ , degeneration  $(\mathbb{A}_1, 8\mathbb{A}_1) \subset 9\mathbb{A}_1$ :

$j$	19	21	21	23 *	23
$H$	$H_{34,1}$	$H_{34,3}$	$H_{34,3}$	$H_{34,1}$	$H_{34,4}$
orbits of	$(\alpha_{2,3}, \alpha_{1,1}),$ $(\alpha_{2,4}, \alpha_{1,1})$	$(\alpha_{1,2}, \alpha_{1,4}),$ $(\alpha_{3,2}, \alpha_{1,4})$	$(\alpha_{2,2}, \alpha_{1,4})$	$(\alpha_5, \alpha_2), (\alpha_6, \alpha_2),$ $(\alpha_{17}, \alpha_2), (\alpha_{21}, \alpha_2)$	$(\alpha_{10}, \alpha_4),$ $(\alpha_{24}, \alpha_4)$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_1$	$A_1$	$3A_1$	$A_1$

$\mathbf{n}=34$ , degeneration  $(\mathbb{A}_1, 12\mathbb{A}_1) \subset 13\mathbb{A}_1$ :

$j$	19	19	19	21	21	21
$H$	$H_{34,2}$	$H_{34,2}$	$H_{34,2}$	$H_{34,1}$	$H_{34,1}$	$H_{34,1}$
orbits of	$(\alpha_{2,3}, \alpha_{1,1})$	$(\alpha_{1,4}, \alpha_{1,1})$	$(\alpha_{2,4}, \alpha_{1,1})$	$(\alpha_{2,1}, \alpha_{1,3})$	$(\alpha_{2,2}, \alpha_{1,3})$	$(\alpha_{1,2}, \alpha_{1,3}),$ $(\alpha_{3,2}, \alpha_{1,3})$
$(S_{N_i}^\perp)^{(2)}$	$A_3$	$A_1 \oplus A_2$	$A_1 \oplus A_2$	$A_3$	$3A_1$	$3A_1$

23	23
$H_{34,2}$	$H_{34,2}$
$(\alpha_6, \alpha_3)$	$(\alpha_{21}, \alpha_3), (\alpha_{24}, \alpha_3)$
$2A_1$	$2A_1$

$\mathbf{n}=34$ , degeneration  $(\mathbb{A}_1, 6\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$ :

$j$	22 *
$H$	$H_{34,1}$
orbits of	$(\alpha_{1,6}, \alpha_{1,2}), (\alpha_{2,6}, \alpha_{1,2}), (\alpha_{1,10}, \alpha_{1,2}), (\alpha_{2,10}, \alpha_{1,2})$
$(S_{N_i}^\perp)^{(2)}$	$A_2$

$\mathbf{n}=34$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	19	21	21	23 *	23
$H$	$H_{34,2}$	$H_{34,2}$	$H_{34,2}$	$H_{34,2}$	$H_{34,3}$
orbits of	$(\alpha_{3,4}, \alpha_{2,1})$	$(\alpha_{1,1}, \alpha_{1,4}),$ $(\alpha_{1,1}, \alpha_{3,4})$	$(\alpha_{1,1}, \alpha_{2,4})$	$(\alpha_4, \alpha_1)$	$(\alpha_4, \alpha_1),$ $(\alpha_4, \alpha_{10})$
$(S_{N_i}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$\{0\}$	$\{0\}$	$3A_1$	$2A_1$

$\mathbf{n}=34$ , degeneration  $(2\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 8\mathbb{A}_1$ :

$j$	21	23 *
$H$	$H_{34,2}$	$H_{34,3}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2})$	$(\alpha_4, \alpha_2)$
$(S_{N_i}^\perp)^{(2)}$	$\{0\}$	$2A_1$

$\mathbf{n}=34$ , degeneration  $(2\mathbb{A}_1, 12\mathbb{A}_1) \subset 14\mathbb{A}_1$ :

$j$	19	21	23 *
$H$	$H_{34,2}$	$H_{34,1}$	$H_{34,2}$
orbits of	$(\alpha_{3,4}, \alpha_{1,1})$	$(\alpha_{1,1}, \alpha_{1,3})$	$(\alpha_4, \alpha_3)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_2$	$A_3$	$3A_1$

$\mathbf{n}=34$ , degeneration  $(3\mathbb{A}_1, 4\mathbb{A}_1) \subset 7\mathbb{A}_1$ :

$j$	19	19	19	21	21	21
$H$	$H_{34,1}$	$H_{34,1}$	$H_{34,2}$	$H_{34,2}$	$H_{34,2}$	$H_{34,3}$
orbits of	$(\alpha_{1,3}, \alpha_{2,1}),$ $(\alpha_{1,4}, \alpha_{2,1})$	$(\alpha_{1,3}, \alpha_{4,1}),$ $(\alpha_{1,4}, \alpha_{4,1})$	$(\alpha_{1,3}, \alpha_{2,1})$	$(\alpha_{2,2}, \alpha_{1,4}),$ $(\alpha_{2,2}, \alpha_{3,4})$	$(\alpha_{2,2}, \alpha_{2,4})$	$(\alpha_{2,1}, \alpha_{2,4})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_2$	$A_1 \oplus A_2$	$A_1 \oplus A_3$	$2A_1$	$2A_1$	$A_3$

23 *	23
$H_{34,2}$	$H_{34,4}$
$(\alpha_2, \alpha_1)$	$(\alpha_2, \alpha_1), (\alpha_2, \alpha_5), (\alpha_3, \alpha_1), (\alpha_3, \alpha_5)$
$3A_1$	$2A_1$

$\mathbf{n}=34$ , degeneration  $(3\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 3\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{34,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=34$ , degeneration  $(3\mathbb{A}_1, 8\mathbb{A}_1) \subset 11\mathbb{A}_1$ :

$j$	19	21	23 *
$H$	$H_{34,1}$	$H_{34,3}$	$H_{34,4}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}), (\alpha_{1,4}, \alpha_{1,1})$	$(\alpha_{2,1}, \alpha_{1,4})$	$(\alpha_2, \alpha_4), (\alpha_3, \alpha_4)$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$A_3$	$2A_1$

$\mathbf{n}=34$ , degeneration  $(3\mathbb{A}_1, 12\mathbb{A}_1) \subset 15\mathbb{A}_1$ :

$j$	19	23 *
$H$	$H_{34,2}$	$H_{34,2}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1})$	$(\alpha_2, \alpha_3)$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$3A_1$

**n=34**, degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	21	23 *	23
$H$	$H_{34,2}$	$H_{34,3}$	$H_{34,4}$
orbits of	$(\alpha_{1,4}, \alpha_{3,4})$	$(\alpha_1, \alpha_{10})$	$(\alpha_1, \alpha_5)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$
$\natural\{v \in S_{N_j}^\perp \mid v^2 = -4 \text{ and } v \perp (S_{N_j}^\perp)^{(2)}\}$	2	6	2

**n=34**, degeneration  $(4\mathbb{A}_1, (6\mathbb{A}_1)_I) \subset 10\mathbb{A}_1$ :

$j$	21	23 *
$H$	$H_{34,3}$	$H_{34,3}$
orbits of	$(\alpha_{2,4}, \alpha_{1,1})$	$(\alpha_1, \alpha_3), (\alpha_{10}, \alpha_3)$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$2A_1$

**n=34**, degeneration  $(4\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 10\mathbb{A}_1$ :

$j$	21	21	23
$H$	$H_{34,2}$	$H_{34,2}$	$H_{34,3}$
orbits of	$(\alpha_{1,4}, \alpha_{1,2}), (\alpha_{3,4}, \alpha_{1,2})$	$(\alpha_{2,4}, \alpha_{1,2})$	$(\alpha_1, \alpha_2), (\alpha_{10}, \alpha_2)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$

**n=34**, degeneration  $(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	19	23 *
$H$	$H_{34,1}$	$H_{34,4}$
orbits of	$(\alpha_{4,1}, \alpha_{1,1})$	$(\alpha_1, \alpha_4), (\alpha_5, \alpha_4)$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$2A_1$

**n=34**, degeneration  $(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$ :

$j$	19	21 *
$H$	$H_{34,1}$	$H_{34,3}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1})$	$(\alpha_{2,4}, \alpha_{1,4})$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$	$A_3$

**n=34**, degeneration  $(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{34,2}$
orbits of	$(\alpha_1, \alpha_3)$
$(S_{N_i}^\perp)^{(2)}$	$3A_1$

**n=34**, degeneration  $(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 4\mathbb{D}_4$ :

$j$	19 *
$H$	$H_{34,2}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1})$
$(S_{N_j}^\perp)^{(2)}$	$A_2 \oplus A_3$

**n=34**, degeneration  $(4\mathbb{A}_1, 6\mathbb{A}_2) \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2$ :

$j$	22 *
$H$	$H_{34,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}), (\alpha_{2,1}, \alpha_{1,2})$
$(S_{N_i}^\perp)^{(2)}$	$2A_2$

**n=34**, degeneration  $((6\mathbb{A}_1)_I, (6\mathbb{A}_1)_{II}) \subset 12\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{34,3}$
orbit of	$(\alpha_3, \alpha_2)$
$(S_{N_i}^\perp)^{(2)}$	$2A_1$

**n=34**, degeneration  $((6\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 14\mathbb{A}_1$ :

$j$	21	23 *
$H$	$H_{34,3}$	$H_{34,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,4})$	$(\alpha_1, \alpha_2), (\alpha_8, \alpha_2)$
$(S_{N_i}^\perp)^{(2)}$	$A_3$	$4A_1$

**n=34**, degeneration  $((6\mathbb{A}_1)_I, 12\mathbb{A}_1) \subset 6\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{34,1}$
orbit of	$(\alpha_{2,3}, \alpha_{1,3})$
$(S_{N_i}^\perp)^{(2)}$	$2A_1 \oplus A_3$

$\mathbf{n}=39$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	$23 *$
$H$	$H_{39,2}$
orbits of	$(\alpha_2, \alpha_3), (\alpha_2, \alpha_4), (\alpha_3, \alpha_4)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=39$ , degeneration  $(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	$23 *$
$H$	$H_{39,2}$
orbits of	$(\alpha_2, \alpha_5), (\alpha_3, \alpha_5), (\alpha_4, \alpha_5)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=39$ , degeneration  $(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	$23 *$
$H$	$H_{39,3}$
orbits of	$(\alpha_3, \alpha_5)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=40$ , degeneration  $(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	$23 *$
$H$	$H_{40,1}$
orbits of	$(\alpha_1, \alpha_3)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=49$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	$23 *$	$23$
$H$	$H_{49,1}$	$H_{49,3}$
orbits of	$(\alpha_2, \alpha_4)$	$(\alpha_2, \alpha_4), (\alpha_2, \alpha_8), (\alpha_2, \alpha_{10}), (\alpha_2, \alpha_{11}), (\alpha_4, \alpha_8),$ $(\alpha_4, \alpha_{10}), (\alpha_4, \alpha_{11}), (\alpha_8, \alpha_{10}), (\alpha_8, \alpha_{11}), (\alpha_{10}, \alpha_{11})$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$A_1$

$\mathbf{n}=49$ , degeneration  $(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	$23 *$
$H$	$H_{49,1}$
orbits of	$(\alpha_2, \alpha_1), (\alpha_4, \alpha_1)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

## 4 Tables of types of degenerations of Kählerian K3 surfaces with symplectic automorphism group $D_8$

Unfortunately, for remaining groups of small order (when  $n < 12$ ), there are many types of degenerations. From our point of view, it is reasonable to consider classification of degenerations for each of these groups separately. The number of cases depends on a group very much. Here, we consider the case of  $D_8$ .

Table 4: Types and lattices  $S$  of degenerations of codimension  $\geq 2$  of Kählerian K3 surfaces with symplectic automorphism group  $D_8$ .

$\mathbf{n}$	$ G $	$i$	$G$	$\text{rk } S_G$	$Deg$	$\text{rk } S$	$q_S$
10	8	3	$D_8$	15	$(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$	17	$4_7^{+5} *$
					$(\mathbb{A}_1, (2\mathbb{A}_1)_I) \subset 3\mathbb{A}_1$	17	$2_7^{+1}, 4_0^{+4}$
					$(\mathbb{A}_1, (2\mathbb{A}_1)_{II}) \subset \mathbb{A}_3$	17	$4_7^{+5} *$
					$(\mathbb{A}_1, 4\mathbb{A}_1) \subset 5\mathbb{A}_1$	17	$2_7^{+1}, 4_1^{+3}, 8_7^{+1}$
					$(\mathbb{A}_1, 8\mathbb{A}_1) \subset 9\mathbb{A}_1$	17	$2_1^{+1}, 4_6^{+4}$
					$(\mathbb{A}_1, 2\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 2\mathbb{A}_2$	17	$2_7^{+1}, 4_{II}^{+4} *$
					$((2\mathbb{A}_1)_I, (2\mathbb{A}_1)_{II}) \subset 4\mathbb{A}_1$	17	$4_7^{+5} *$
					$((2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 6\mathbb{A}_1)_I$	17	$2_6^{+2}, 4_0^{+2}, 8_1^{+1}$
					$((2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 6\mathbb{A}_1)_{II}$	17	$2_6^{+2}, 4_{II}^{+2}, 8_1^{+1}$
					$((2\mathbb{A}_1)_{II}, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$	17	$2_{II}^{+2}, 4_{II}^{+2}, 8_7^{+1}$
					$((2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 2\mathbb{A}_3$	17	$2_0^{+2}, 4_7^{+3} *$
					$((2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$	17	$2_2^{+2}, 4_5^{+3} *$
					$((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_I$	17	$4_7^{+1}, 8_0^{+2} *$
					$((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_{II}$	17	$2_{II}^{+2}, 4_7^{+3} *$
					$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 4\mathbb{A}_2$	17	$4_1^{-3}, 3^{+1} *$
					$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$	17	$4_6^{+2}, 8_1^{+1} *$
					$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$	17	$4_7^{+3} *$
					$(4\mathbb{A}_1, 2\mathbb{A}_2) \subset 4\mathbb{A}_1 \amalg 2\mathbb{A}_2$	17	$4_{II}^{-2}, 8_3^{-1} *$
					$(\mathbb{A}_1, \mathbb{A}_1, (2\mathbb{A}_1)_I) \subset 4\mathbb{A}_1$	18	$4_6^{+4} *$
					$(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$	18	$4_7^{+3}, 8_7^{+1} *$
					$(\mathbb{A}_1, \mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$	18	$4_6^{+4} *$
					$(\mathbb{A}_1, (2\mathbb{A}_1)_{II}, (2\mathbb{A}_1)_I) \subset \mathbb{A}_3 \amalg 2\mathbb{A}_1$	18	$4_6^{+4} *$
					$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 7\mathbb{A}_1$	18	$2_7^{+1}, 4_6^{+2}, 8_5^{-1}$
					$(\mathbb{A}_1, (2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 2\mathbb{A}_3$	18	$2_7^{+1}, 4_7^{+3} *$
					$(\mathbb{A}_1, (2\mathbb{A}_1)_{II}, 4\mathbb{A}_1) \subset \mathbb{A}_3 \amalg 4\mathbb{A}_1$	18	$4_5^{+3}, 8_1^{+1}$
					$(\mathbb{A}_1, (2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 11\mathbb{A}_1$	18	$2_7^{+1}, 4_7^{+3} *$
					$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 9\mathbb{A}_1$	18	$2_7^{+1}, 4_7^{+1}, 8_4^{-2}$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{A}_2$	18	$2_7^{+1}, 4_1^{+3}, 3^{+1} *$



<b>n</b>	$G$	$\text{rk } S_G$	$Deg$	$\text{rk } S$	$q_S$
10	$D_8$	15	$(\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 13\mathbb{A}_1$	18	$2_7^{+1}, 4_0^{+2}, 8_7^{+1}$
			$(\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$2_7^{+1}, 4_7^{+3}$
			$(\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset 5\mathbb{A}_1 \amalg 2\mathbb{A}_2$	18	$2_3^{-1}, 4_{II}^{+2}, 8_3^{-1}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$	18	$4_5^{+3}, 8_1^{+1}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_{II} \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$	18	$4_7^{+3}, 8_7^{+1} *$
			$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_3$	18	$4_6^{+4} *$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	18	$2_6^{+2}, 8_0^{+2}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	18	$4_6^{+4}$
			$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	18	$2_{II}^{+2}, 8_6^{+2}$
			$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	18	$4_6^{+4}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_2$	18	$2_2^{+2}, 4_6^{+2}, 3^{+1} *$
			$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_2$	18	$2_{II}^{+2}, 4_{II}^{-2}, 3^{+1}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_1$	18	$2_0^{+2}, 4_7^{+1}, 8_3^{-1}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_5$	18	$2_2^{-2}, 4_{II}^{+2} *$

<b>n</b>	$G$	$\text{rk } S_G$	$Deg$	$\text{rk } S$	$q_S$
10	$D_8$	15	$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1$	18	$2_2^{-2}, 4_1^{+1}, 8_3^{-1} *$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$2_2^{-2}, 4_0^{+2}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$2_2^{-2}, 4_{II}^{-2} *$
			$((2\mathbb{A}_1)_I, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 8\mathbb{A}_1$	18	$2_4^{-2}, 4_6^{-2} *$
			$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1$	18	$2_{II}^{-2}, 4_1^{+1}, 8_5^{-1} *$
			$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_2$	18	$4_7^{+1}, 8_5^{-1}, 3^{+1} *$
			$\begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3$	18	$2_{II}^{-2}, 4_2^{-2} *$
			$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1$	18	$2_{II}^{+2}, 4_6^{+2} *$
			$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$4_5^{-1}, 8_5^{-1} *$
			$\begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{D}_4$	18	$4_6^{+2} *$
			$(4\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset (8\mathbb{A}_1)_I \amalg 2\mathbb{A}_2$	18	$8_6^{+2} *$
			$(4\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset 4\mathbb{A}_2 \amalg 2\mathbb{A}_2$	18	$4_{II}^{-2}, 3^{+1} *$

<b>n</b>	<b>G</b>	<b>Deg</b>	<b>rk S</b>	<b>q<sub>S</sub></b>
10	$D_8$	$\begin{pmatrix} \mathbb{A}_1 & 2\mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1} *$
		$\begin{pmatrix} \mathbb{A}_1 & 2\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	19	$4_5^{-1}, 8_4^{-2} *$
		$(\mathbb{A}_1, \mathbb{A}_1, (2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$	19	$4_5^{+3} *$
		$(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 14\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1}$
		$(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$4_5^{+3} *$
		$\begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_3 \amalg 6\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1}$
		$\begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_3$	19	$4_5^{+3} *$
		$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 2\mathbb{A}_3 \amalg 4\mathbb{A}_1$	19	$2_3^{-1}, 4_5^{-1}, 8_1^{+1} *$
		$\begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_3 \amalg 8\mathbb{A}_1$	19	$4_7^{+1}, 8_6^{+2}$
		$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_3^{-1}, 4_0^{+2}, 3^{+1} *$
		$(\mathbb{A}_1, (2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset \mathbb{A}_3 \amalg 4\mathbb{A}_2$	19	$4_7^{-3}, 3^{+1} *$
		$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 15\mathbb{A}_1$	19	$2_1^{+1}, 4_5^{-1}, 8_3^{-1} *$

<b>n</b>	<b>G</b>	<b>Deg</b>	<b>rk S</b>	<b>q<sub>S</sub></b>
10	$D_8$	$(\mathbb{A}_1, (2\mathbb{A}_1)_I, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 2\mathbb{A}_3, \amalg 8\mathbb{A}_1$	19	$2_3^{-1}, 4_6^{-2} *$
		$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_5^{-1}, 4_4^{-2}$
		$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_1^{+1}, 4_7^{+1}, 8_3^{-1}, 3^{+1}$
		$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_7^{+1}, 4_5^{-1}, 8_5^{-1} *$
		$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 4\mathbb{D}_4$	19	$2_7^{+1}, 4_6^{+2}$
		$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & \mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & & 2\mathbb{A}_2 \end{pmatrix} \subset 9\mathbb{A}_1 \amalg 2\mathbb{A}_2$	19	$2_7^{+1}, 8_6^{+2}$
		$(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$2_7^{+1}, 4_{II}^{+2}, 3^{+1} *$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1$	19	$4_7^{+1}, 8_6^{+2}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1$	19	$4_5^{+3}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$4_7^{-3}, 3^{+1} *$

<b>n</b>	<b>G</b>	<b>Deg</b>	<b>rk S</b>	<b>q<sub>S</sub></b>
10	$D_8$	$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 2\mathbb{A}_3$	19	$4_6^{+2}, 8_7^{+1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_5$	19	$4_5^{+3} *$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1$	19	$4_6^{+2}, 8_7^{+1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_2^{-2}, 8_1^{+1}, 3^{+1} *$
		$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_{II}^{+2}, 8_3^{-1}, 3^{+1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$4_5^{+3} *$
		$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$4_5^{+3} *$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_2$	19	$2_0^{+2}, 4_7^{+1}, 3^{+1} *$

<b>n</b>	<b>G</b>	<b>Deg</b>	<b>rk S</b>	<b>q<sub>S</sub></b>
10	$D_8$	$\begin{pmatrix} 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 2\mathbb{A}_5$	19	$2_6^{+2}, 8_7^{+1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_2^{-2}, 8_3^{-1} *$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_3$	19	$2_4^{-2}, 4_5^{-1} *$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{D}_4$	19	$2_6^{+2}, 4_7^{+1} *$
		$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1$	19	$4_5^{+3} *$
		$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_{II}^{-2}, 8_5^{-1} *$
		$\begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_2$	19	$4_1^{+1}, 3^{+2} *$
		$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & & 2\mathbb{A}_2 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$8_3^{-1}, 3^{+1} *$

Table 5: Markings  $S \subset N_j$  by Niemeier lattices, and lattices  $S_{N_j}^\perp$  of degenerations of codimension  $\geq 2$  of Kählerian K3 surfaces with symplectic automorphism groups  $D_8$  in notations [18], [19], [20].

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$ :

$j$	21	23	23 *
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1})$	$(\alpha_1, \alpha_{24}), (\alpha_5, \alpha_6)$	$(\alpha_1, \alpha_6)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$\{0\}$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, (2\mathbb{A}_1)_I) \subset 3\mathbb{A}_1$ :

$j$	19	21	21	23
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}), (\alpha_{3,4}, \alpha_{1,4})$	$(\alpha_{2,2}, \alpha_{2,4})$	$(\alpha_{2,2}, \alpha_{2,4})$	$(\alpha_5, \alpha_{10}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$3A_1$	$3A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, (2\mathbb{A}_1)_{II}) \subset \mathbb{A}_3$ :

$j$	18	21 *
$H$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{2,5}, \alpha_{1,5})$	$(\alpha_{2,2}, \alpha_{1,2})$
$(S_{N_j}^\perp)^{(2)}$	$\{0\}$	$2A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1) \subset 5\mathbb{A}_1$ :

$j$	18	18	18	18
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{2,5}, \alpha_{1,1}), \dots$	$(\alpha_{4,5}, \alpha_{1,1}), \dots$	$(\alpha_{2,5}, \alpha_{2,1}), \dots$	$(\alpha_{4,5}, \alpha_{2,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1$	$A_1$	$A_1$	$A_1$

19	19	19	19	21
$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
$(\alpha_{1,3}, \alpha_{2,1}), \dots$	$(\alpha_{2,3}, \alpha_{2,1}), \dots$	$(\alpha_{1,3}, \alpha_{4,1}), \dots$	$(\alpha_{2,3}, \alpha_{4,1}), \dots$	$(\alpha_{1,1}, \alpha_{2,3}), \dots$
$A_1 \oplus A_3$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$3A_1$

21	21	21	21	21	21
$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
$(\alpha_{2,1}, \alpha_{2,3})$	$(\alpha_{2,2}, \alpha_{2,3})$	$(\alpha_{1,1}, \alpha_{1,4}), \dots$	$(\alpha_{2,1}, \alpha_{1,4})$	$(\alpha_{2,2}, \alpha_{1,4})$	$(\alpha_{2,1}, \alpha_{1,3}), \dots$
$3A_1$	$A_1 \oplus A_3$	$3A_1$	$3A_1$	$A_1 \oplus A_3$	$3A_1$

21	21	21	21	21	22
$H_{10,2}$	$H_{10,2}$	$H_{10,2}$	$H_{10,2}$	$H_{10,2}$	$H_{10,1}$
$(\alpha_{2,2}, \alpha_{1,3}), \dots$	$(\alpha_{2,1}, \alpha_{2,3})$	$(\alpha_{2,2}, \alpha_{2,3})$	$(\alpha_{2,1}, \alpha_{1,4})$	$(\alpha_{2,2}, \alpha_{1,4}), \dots$	$(\alpha_{1,1}, \alpha_{1,3}), \dots$
$3A_1$	$3A_1$	$3A_1$	$3A_1$	$3A_1$	$A_2$

22	23	23
$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
$(\alpha_{2,1}, \alpha_{1,3}), \dots$	$(\alpha_1, \alpha_3), \dots$	$(\alpha_1, \alpha_3), \dots$
$A_2$	$3A_1$	$A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, 8\mathbb{A}_1) \subset 9\mathbb{A}_1$ :

$j$	19	19	21	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{1,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,3}), \dots$	$(\alpha_{2,1}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$3A_1$	$3A_1$

21	23
$H_{10,1}$	$H_{10,1}$
$(\alpha_{2,2}, \alpha_{1,3})$	$(\alpha_1, \alpha_2), \dots$
$A_1 \oplus A_3$	$3A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, 2\mathbb{A}_2) \subset \mathbb{A}_1 \amalg \mathbb{A}_2$ :

$j$	22 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,9}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_2$

$\mathbf{n}=10$ , degeneration  $((2\mathbb{A}_1)_I, (2\mathbb{A}_1)_{II}) \subset 4\mathbb{A}_1$ :



$j$	18	21 *	21	23
$H$	$H_{10,1}$	$H_{10,2}$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{3,1}, \alpha_{1,5}), \dots$	$(\alpha_{1,1}, \alpha_{1,2})$	$(\alpha_{2,4}, \alpha_{1,2})$	$(\alpha_{10}, \alpha_2)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$\{0\}$	$2A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $((2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	18	18	19	19
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{3,3}, \alpha_{1,1}), \dots$	$(\alpha_{3,3}, \alpha_{2,1}), \dots$	$(\alpha_{3,3}, \alpha_{2,1}), \dots$	$(\alpha_{3,3}, \alpha_{4,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$	$2A_1 \oplus A_3$	$2A_1 \oplus A_3$

21	21	21	21	21	21
$H_{10,1}$	$H_{10,1}$	$H_{10,2}$	$H_{10,2}$	$H_{10,2}$	$H_{10,2}$
$(\alpha_{1,2}, \alpha_{1,4})$	$(\alpha_{2,4}, \alpha_{2,3})$	$(\alpha_{1,1}, \alpha_{1,3}), \dots$	$(\alpha_{1,1}, \alpha_{2,3})$	$(\alpha_{1,4}, \alpha_{1,3}), \dots$	$(\alpha_{2,4}, \alpha_{2,3})$
$A_3$	$2A_1 \oplus A_3$	$2A_1$	$2A_1$	$4A_1$	$4A_1$

23	23
$H_{10,1}$	$H_{10,2}$
$(\alpha_{10}, \alpha_3), \dots$	$(\alpha_{16}, \alpha_3), \dots$
$4A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $((2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 6\mathbb{A}_1)_{II}$ :

$j$	18	21	21	23
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{3,1}, \alpha_{1,1}), \dots$	$(\alpha_{1,2}, \alpha_{2,3})$	$(\alpha_{1,1}, \alpha_{1,4})$	$(\alpha_{10}, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$	$2A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $((2\mathbb{A}_1)_{II}, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	18	18	21	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{1,5}, \alpha_{1,1}), \dots$	$(\alpha_{1,5}, \alpha_{2,1}), \dots$	$(\alpha_{1,2}, \alpha_{1,3}), \dots$	$(\alpha_{1,2}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$	$2A_1$

21	23
$H_{10,2}$	$H_{10,2}$
$(\alpha_{1,2}, \alpha_{1,4})$	$(\alpha_2, \alpha_3), \dots$
$2A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $((2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 2\mathbb{A}_3$ :

$j$	18	21 *	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{3,1}, \alpha_{2,1}), \dots$	$(\alpha_{2,4}, \alpha_{1,4})$	$(\alpha_{2,4}, \alpha_{1,4})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$2A_1 \oplus A_3$	$4A_1$

$\mathbf{n}=10$ , degeneration  $((2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$ :

$j$	19	21	21	23 *
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{3,3}, \alpha_{1,1}), \dots$	$(\alpha_{1,2}, \alpha_{1,3})$	$(\alpha_{2,4}, \alpha_{1,3})$	$(\alpha_{10}, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_3$	$A_3$	$2A_1 \oplus A_3$	$4A_1$

$\mathbf{n}=10$ , degeneration  $((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_I$ :

$j$	18	18	18	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3})$	$(\alpha_{1,1}, \alpha_{2,3}), \dots$	$(\alpha_{2,1}, \alpha_{2,3})$	$(\alpha_{2,3}, \alpha_{1,4})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$	$A_3$	$2A_1 \oplus A_3$

21	21	22	23 *
$H_{10,2}$	$H_{10,2}$	$H_{10,1}$	$H_{10,2}$
$(\alpha_{1,3}, \alpha_{1,4}), \dots$	$(\alpha_{2,3}, \alpha_{1,4})$	$(\alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_3, \alpha_4), \dots$
$4A_1$	$4A_1$	$2A_2$	$2A_1$

$\mathbf{n}=10$ , degeneration  $((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_{II}$ :

$j$	21	23	23 *
$H$	$H_{10,2}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3})$	$(\alpha_3, \alpha_7)$	$(\alpha_3, \alpha_5), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$4A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 4\mathbb{A}_2$ :

$j$	18	19 *	21	22
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{2,1}, \alpha_{4,1})$	$(\alpha_{1,3}, \alpha_{2,3}), \dots$	$(\alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$2A_3$	$4A_1$	$2A_2$

$\mathbf{n}=10$ , degeneration  $(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	19	21	23 *
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{4,1}, \alpha_{1,1})$	$(\alpha_{1,4}, \alpha_{1,3})$	$(\alpha_3, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_3$	$2A_1 \oplus A_3$	$4A_1$

$\mathbf{n}=10$ , degeneration  $(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$ :

$j$	19	21 *
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{2,1}, \alpha_{1,1})$	$(\alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_3$	$2A_1 \oplus A_3$

$\mathbf{n}=10$ , degeneration  $(4\mathbb{A}_1, 2\mathbb{A}_2) \subset 4\mathbb{A}_1 \amalg 2\mathbb{A}_2$ :

$j$	22 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,9}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, (2\mathbb{A}_1)_I) \subset 4\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{10,1}$
orbits of	$(\alpha_5, \alpha_6, \alpha_{10}), (\alpha_1, \alpha_{24}, \alpha_{15})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	21	21	23	23 *
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{2,3})$	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{1,4})$	$(\alpha_1, \alpha_{24}, \alpha_3), \dots$	$(\alpha_1, \alpha_6, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$	$\{0\}$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$ :

$j$	21	23
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{1,3})$	$(\alpha_1, \alpha_{24}, \alpha_2), (\alpha_5, \alpha_6, \alpha_2)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, (2\mathbb{A}_1)_{II}, (2\mathbb{A}_1)_I) \subset \mathbb{A}_3 \amalg 2\mathbb{A}_1$ :

$j$	21 *
$H$	$H_{10,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,4})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=10$ , degeneration  $\left( \begin{array}{ccc} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{array} \right) \subset 7\mathbb{A}_1$ :

$j$	19	19	21	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{2,1}), \dots$	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{4,1}), \dots$	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{2,3})$	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$3A_1$

21	23
$H_{10,2}$	$H_{10,1}$
$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,3}), \dots$	$(\alpha_5, \alpha_{10}, \alpha_3), \dots$
$3A_1$	$3A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, (2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 2\mathbb{A}_3$ :

$j$	21 *	21
$H$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,4})$	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,4})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$3A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, (2\mathbb{A}_1)_{II}, 4\mathbb{A}_1) \subset \mathbb{A}_3 \amalg 4\mathbb{A}_1$ :

$j$	18	18	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{2,5}, \alpha_{1,5}, \alpha_{1,1}), \dots$	$(\alpha_{2,5}, \alpha_{1,5}, \alpha_{2,1}), \dots$	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$\{0\}$	$\{0\}$	$2A_1$

21	21
$H_{10,2}$	$H_{10,2}$
$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,3})$	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{1,4})$
$2A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, (2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 11\mathbb{A}_1$ :

$j$	19	21	23 *
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{1,1}), \dots s$	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,3})$	$(\alpha_5, \alpha_{10}, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$3A_1$

$\mathbf{n}=10$ , degeneration  $\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 9\mathbb{A}_1$ :

$j$	18	18	18	18
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{2,5}, \alpha_{1,1}, \alpha_{1,3})$	$(\alpha_{4,5}, \alpha_{1,1}, \alpha_{1,3})$	$(\alpha_{2,5}, \alpha_{1,1}, \alpha_{2,3}), \dots$	$(\alpha_{4,5}, \alpha_{1,1}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1$	$A_1$	$A_1$	$A_1$

18	18	21	21	21 *
$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
$(\alpha_{2,5}, \alpha_{2,1}, \alpha_{2,3})$	$(\alpha_{4,5}, \alpha_{2,1}, \alpha_{2,3})$	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{1,4}), \dots$	$(\alpha_{2,1}, \alpha_{2,3}, \alpha_{1,4})$	$(\alpha_{2,2}, \alpha_{2,3}, \alpha_{1,4})$
$A_1$	$A_1$	$3A_1$	$3A_1$	$A_1 \oplus A_3$

21	21	21	21
$H_{10,2}$	$H_{10,2}$	$H_{10,2}$	$H_{10,2}$
$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{1,4}), \dots$	$(\alpha_{2,2}, \alpha_{1,3}, \alpha_{1,4}), \dots$	$(\alpha_{2,1}, \alpha_{2,3}, \alpha_{1,4})$	$(\alpha_{2,2}, \alpha_{2,3}, \alpha_{1,4})$
$3A_1$	$3A_1$	$3A_1$	$3A_1$

22	22	22	23
$H_{10,1}$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{1,5}), \dots$	$(\alpha_1, \alpha_3, \alpha_4), \dots$
$A_2$	$A_2$	$A_2$	$A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{A}_2$ :

$j$	18	18	19
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{2,5}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{4,5}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{1,3}, \alpha_{2,1}, \alpha_{4,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1$	$A_1$	$A_1 \oplus A_3$

19	21	21	22 *
$H_{10,1}$	$H_{10,2}$	$H_{10,2}$	$H_{10,1}$
$(\alpha_{2,3}, \alpha_{2,1}, \alpha_{4,1}), \dots$	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{2,3}), \dots$	$(\alpha_{2,2}, \alpha_{1,3}, \alpha_{2,3}), \dots$	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$A_1 \oplus A_3$	$3A_1$	$3A_1$	$A_2$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 13\mathbb{A}_1$ :

$j$	19	19	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{4,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{4,1}, \alpha_{1,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,4}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$3A_1$

21	21	23
$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
$(\alpha_{2,1}, \alpha_{1,4}, \alpha_{1,3})$	$(\alpha_{2,2}, \alpha_{1,4}, \alpha_{1,3})$	$(\alpha_1, \alpha_3, \alpha_2), \dots$
$3A_1$	$A_1 \oplus A_3$	$3A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{A}_3$ :

$j$	19	19	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$	$3A_1$

21	21
$H_{10,1}$	$H_{10,1}$
$(\alpha_{2,1}, \alpha_{2,3}, \alpha_{1,3})$	$(\alpha_{2,2}, \alpha_{2,3}, \alpha_{1,3})$
$3A_1$	$A_1 \oplus A_3$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset 5\mathbb{A}_1 \amalg 2\mathbb{A}_2$ :

$j$	22	22
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{1,9})$	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{1,9})$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_2$

$\mathbf{n}=10$ , degeneration  $\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$ :

$j$	18	18	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{3,3}, \alpha_{1,5}, \alpha_{1,1}), \dots$	$(\alpha_{3,3}, \alpha_{1,5}, \alpha_{2,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$\{0\}$

21	21	23
$H_{10,2}$	$H_{10,2}$	$H_{10,2}$
$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,3})$	$(\alpha_{2,4}, \alpha_{1,2}, \alpha_{1,3}), \dots$	$(\alpha_{16}, \alpha_2, \alpha_3), \dots$
$\{0\}$	$2A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_{II} \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$ :

$j$	18	21 *	21	23
$H$	$H_{10,1}$	$H_{10,2}$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{3,1}, \alpha_{1,5}, \alpha_{1,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,4})$	$(\alpha_{2,4}, \alpha_{1,2}, \alpha_{2,3})$	$(\alpha_{10}, \alpha_2, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$\{0\}$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,2}$
orbits of	$(\alpha_{1,2}, \alpha_{2,4}, \alpha_{1,4})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1:$$

$j$	18	18	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{3,1}, \alpha_{1,3}, \alpha_{1,1}), \dots$	$(\alpha_{3,1}, \alpha_{2,3}, \alpha_{1,1}), \dots$	$(\alpha_{1,2}, \alpha_{1,4}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$	$A_3$

21	21	23
$H_{10,2}$	$H_{10,2}$	$H_{10,2}$
$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{1,4}), \dots$	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{1,4})$	$(\alpha_{10}, \alpha_4, \alpha_3)$
$2A_1$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1:$$

$j$	21	23
$H$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{3,3})$	$(\alpha_{16}, \alpha_3, \alpha_5), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1:$$



$j$	18	18	18
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,5}, \alpha_{1,1}, \alpha_{1,3})$	$(\alpha_{1,5}, \alpha_{1,1}, \alpha_{2,3}), \dots$	$(\alpha_{1,5}, \alpha_{2,1}, \alpha_{2,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$

21	21	23
$H_{10,2}$	$H_{10,2}$	$H_{10,2}$
$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{1,4}), \dots$	$(\alpha_{1,2}, \alpha_{2,3}, \alpha_{1,4})$	$(\alpha_2, \alpha_3, \alpha_4), \dots$
$2A_1$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1:$$

$j$	21	23
$H$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{3,3})$	$(\alpha_2, \alpha_3, \alpha_5), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_2$$

$j$	18	19	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{3,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{3,3}, \alpha_{2,1}, \alpha_{4,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$2A_1 \oplus A_3$	$2A_1$

21 *
$H_{10,2}$
$(\alpha_{2,4}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$4A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_2:$$

$j$	18	21
$H$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,5}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_i}^\perp)^{(2)}$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_1:$$

$j$	18	18	21 *
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{3,3}, \alpha_{2,3}, \alpha_{1,1}), \dots$	$(\alpha_{3,1}, \alpha_{2,1}, \alpha_{2,3}), \dots$	$(\alpha_{2,4}, \alpha_{1,4}, \alpha_{2,3})$
$(S_{N_i}^\perp)^{(2)}$	$A_3$	$A_3$	$2A_1 \oplus A_3$

21	21
$H_{10,2}$	$H_{10,2}$
$(\alpha_{2,4}, \alpha_{1,4}, \alpha_{1,3}), \dots$	$(\alpha_{2,4}, \alpha_{1,4}, \alpha_{2,3})$
$4A_1$	$4A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_5:$$

$j$	18 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{3,1}, \alpha_{2,1}, \alpha_{1,1}), \dots$
$(S_{N_i}^\perp)^{(2)}$	$A_3$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1:$$

$j$	19	21	23 *
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{3,3}, \alpha_{4,1}, \alpha_{1,1}), \dots$	$(\alpha_{1,2}, \alpha_{1,4}, \alpha_{1,3})$	$(\alpha_{10}, \alpha_3, \alpha_2), \dots$
$(S_{N_i}^\perp)^{(2)}$	$2A_1 \oplus A_3$	$A_3$	$4A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	19	21
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{3,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,4}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_3$	$2A_1 \oplus A_3$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,2}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$

$$\mathbf{n}=10, \text{ degeneration } ((2\mathbb{A}_1)_I, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 8\mathbb{A}_1:$$

$j$	21 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{2,4}, \alpha_{1,4}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_3$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1:$$

$j$	21	23 *
$H$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{1,3}, \alpha_{1,4}, \alpha_{3,3})$	$(\alpha_3, \alpha_4, \alpha_5), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_2:$$

$j$	18	18	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{2,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{1,4}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$	$4A_1$

22 *
$H_{10,1}$
$(\alpha_{1,5}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$2A_2$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,2}$
orbits of	$(\alpha_{1,3}, \alpha_{2,3}, \alpha_{3,3})$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{10,1}$
orbits of	$(\alpha_3, \alpha_7, \alpha_2)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,4}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_3$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{D}_4:$$

$j$	19 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{4,1}, \alpha_{2,1}, \alpha_{1,1})$
$(S_{N_j}^\perp)^{(2)}$	$2A_3$

$$\mathbf{n}=10, \text{ degeneration } (4\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset (8\mathbb{A}_1)_I \amalg 2\mathbb{A}_2:$$

$j$	22 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,5}, \alpha_{1,9})$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$

$$\mathbf{n}=10, \text{ degeneration } (4\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset 4\mathbb{A}_2 \amalg 2\mathbb{A}_2:$$

$j$	22 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{2,3}, \alpha_{1,9}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 2\mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{10,1}$
orbits of	$(\alpha_5, \alpha_6, \alpha_{10}, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 2\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1:$$

$j$	21 *	23
$H$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{2,3}, \alpha_{1,4})$	$(\alpha_1, \alpha_6, \alpha_3, \alpha_4), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$\{0\}$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, (2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{10,1}$
orbits of	$(\alpha_5, \alpha_6, \alpha_{10}, \alpha_2), (\alpha_1, \alpha_{24}, \alpha_{15}, \alpha_2)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 14\mathbb{A}_1$ :

$j$	21	23
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{3,1}, \alpha_{1,4}, \alpha_{1,3})$	$(\alpha_1, \alpha_{24}, \alpha_3, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=10$ , degeneration  $\left( \begin{array}{cccc} \mathbb{A}_1 & \mathbb{A}_3 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{array} \right) \subset \mathbb{A}_3 \amalg 6\mathbb{A}_1$ :

$j$	21	21
$H$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,4}, \alpha_{1,3}), (\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,4}, \alpha_{3,3})$	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,3}, \alpha_{2,4})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,4}, \alpha_{1,4})$
$(S_{N_i}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 2\mathbb{A}_3 \amalg 4\mathbb{A}_1:$$

$j$	21 *	21	21
$H$	$H_{10,1}$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,4}, \alpha_{2,3})$	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,4}, \alpha_{1,3}), \dots$	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,4}, \alpha_{2,3})$
$(S_{N_i}^\perp)^{(2)}$	$A_1 \oplus A_3$	$3A_1$	$3A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_3 \amalg 8\mathbb{A}_1:$$

$j$	18	18	18
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{2,5}, \alpha_{1,5}, \alpha_{1,3}, \alpha_{1,1})$	$(\alpha_{2,5}, \alpha_{1,5}, \alpha_{2,3}, \alpha_{1,1}), \dots$	$(\alpha_{2,5}, \alpha_{1,5}, \alpha_{2,3}, \alpha_{2,1})$
$(S_{N_i}^\perp)^{(2)}$	$\{0\}$	$\{0\}$	$\{0\}$

21	21
$H_{10,2}$	$H_{10,2}$
$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{1,3}, \alpha_{1,4}), \dots$	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,3}, \alpha_{1,4})$
$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_2:$$

$j$	19	21 *
$H$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{2,1}, \alpha_{4,1}), \dots$	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_i}^\perp)^{(2)}$	$A_1 \oplus A_3$	$3A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, (2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset \mathbb{A}_3 \amalg 4\mathbb{A}_2$ :

$j$	18	21 *
$H$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{2,5}, \alpha_{1,5}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_i}^\perp)^{(2)}$	$\{0\}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 15\mathbb{A}_1:$$

$j$	19	23 *
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{4,1}, \alpha_{1,1}), \dots$	$(\alpha_5, \alpha_{10}, \alpha_3, \alpha_2), \dots$
$(S_{N_i}^\perp)^{(2)}$	$A_1 \oplus A_3$	$3A_1$

$\mathbf{n}=10$ , degeneration  $(\mathbb{A}_1, (2\mathbb{A}_1)_I, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 2\mathbb{A}_3, \amalg 8\mathbb{A}_1$ :

$j$	21 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{1,4}, \alpha_{1,3})$
$(S_{N_i}^\perp)^{(2)}$	$A_1 \oplus A_3$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	19	21
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{3,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,2}, \alpha_{2,4}, \alpha_{2,3}, \alpha_{1,3}), \dots$
$(S_{N_i}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$



$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_1 \amalg 4\mathbb{A}_2:$$

$j$	18	18	18
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{2,5}, \alpha_{1,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{4,5}, \alpha_{1,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,5}, \alpha_{2,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1$	$A_1$	$A_1$

18	21	21
$H_{10,1}$	$H_{10,2}$	$H_{10,2}$
$(\alpha_{4,5}, \alpha_{2,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,1}, \alpha_{1,4}, \alpha_{1,3}, \alpha_{2,3}), \dots$	$(\alpha_{2,2}, \alpha_{1,4}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$A_1$	$3A_1$	$3A_1$

22	22
$H_{10,1}$	$H_{10,1}$
$(\alpha_{1,1}, \alpha_{1,5}, \alpha_{1,3}, \alpha_{2,3}), \dots$	$(\alpha_{2,1}, \alpha_{1,5}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$A_2$	$A_2$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21	21	21 *
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,4}, \alpha_{2,3}, \alpha_{1,3}), \dots$	$(\alpha_{2,1}, \alpha_{1,4}, \alpha_{2,3}, \alpha_{1,3})$	$(\alpha_{2,2}, \alpha_{1,4}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$3A_1$	$3A_1$	$A_1 \oplus A_3$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 4\mathbb{D}_4:$$

$j$	19	19
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{4,1}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{4,1}, \alpha_{2,1}, \alpha_{1,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_1 \oplus A_3$	$A_1 \oplus A_3$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & \mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & & 2\mathbb{A}_2 \end{pmatrix} \subset 9\mathbb{A}_1 \amalg 2\mathbb{A}_2:$$

$j$	22	22	22
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{1,5}, \alpha_{1,9}), \dots$	$(\alpha_{2,1}, \alpha_{1,3}, \alpha_{1,5}, \alpha_{1,9}), \dots$	$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{1,5}, \alpha_{1,9}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_2$	$A_2$	$A_2$

$$\mathbf{n}=10, \text{ degeneration } (\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2:$$

$j$	22 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{2,3}, \alpha_{1,9}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_2$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1:$$

$j$	18	18	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{3,1}, \alpha_{1,5}, \alpha_{1,3}, \alpha_{1,1}), \dots$	$(\alpha_{3,1}, \alpha_{1,5}, \alpha_{2,3}, \alpha_{1,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3}, \alpha_{1,4}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$\{0\}$

21	23
$H_{10,2}$	$H_{10,2}$
$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,3}, \alpha_{1,4})$	$(\alpha_{10}, \alpha_2, \alpha_4, \alpha_3), \dots$
$\{0\}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{10,2}$
orbits of	$(\alpha_{16}, \alpha_2, \alpha_3, \alpha_5), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_2:$$

$j$	18	21 *	21
$H$	$H_{10,1}$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{1,5}, \alpha_{3,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3}, \alpha_{2,3}), \dots$	$(\alpha_{2,4}, \alpha_{1,2}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$\{0\}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 2\mathbb{A}_3:$$

$j$	18	18
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,5}, \alpha_{1,1}, \alpha_{3,3}, \alpha_{2,3}), \dots$	$(\alpha_{1,5}, \alpha_{2,3}, \alpha_{3,1}, \alpha_{2,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

21	21
$H_{10,2}$	$H_{10,2}$
$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{2,4}, \alpha_{1,4}), \dots$	$(\alpha_{1,2}, \alpha_{2,3}, \alpha_{2,4}, \alpha_{1,4})$
$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_5:$$

$j$	18 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,5}, \alpha_{3,1}, \alpha_{2,1}, \alpha_{1,1}), (\alpha_{1,5}, \alpha_{3,3}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1:$$

$j$	21	23
$H$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{1,1}, \alpha_{1,4}, \alpha_{1,3}, \alpha_{3,3})$	$(\alpha_{16}, \alpha_4, \alpha_5, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1:$$

$j$	21	23
$H$	$H_{10,2}$	$H_{10,2}$
orbits of	$(\alpha_{1,2}, \alpha_{1,4}, \alpha_{1,3}, \alpha_{3,3})$	$(\alpha_2, \alpha_4, \alpha_3, \alpha_5), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_2:$$

$j$	18 *	21
$H$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{3,3}, \alpha_{1,3}, \alpha_{1,1}, \alpha_{2,1}), \dots$	$(\alpha_{1,1}, \alpha_{1,4}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_2:$$

$j$	18	18	21
$H$	$H_{10,1}$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{1,5}, \alpha_{1,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{1,5}, \alpha_{2,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{1,2}, \alpha_{1,4}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,2}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}, \alpha_{2,3}, \alpha_{3,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,2}$
orbits of	$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{2,3}, \alpha_{3,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_2:$$

$j$	18	21 *
$H$	$H_{10,1}$	$H_{10,2}$
orbits of	$(\alpha_{3,3}, \alpha_{2,3}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,4}, \alpha_{1,4}, \alpha_{1,3}, \alpha_{2,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$4A_1$

$$\mathbf{n}=10, \text{ degeneration } \begin{pmatrix} 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 2\mathbb{A}_5:$$

$j$	18	18
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{3,1}, \alpha_{2,1}, \alpha_{1,1}), \dots$	$(\alpha_{2,3}, \alpha_{3,1}, \alpha_{2,1}, \alpha_{1,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$A_3$

$$\mathbf{n}=10, \text{ degeneration} \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,2}, \alpha_{1,4}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$

$$\mathbf{n}=10, \text{ degeneration} \begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{2,4}, \alpha_{1,4}, \alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_3$

$$\mathbf{n}=10, \text{ degeneration} \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{D}_4:$$

$j$	19 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{3,3}, \alpha_{4,1}, \alpha_{2,1}, \alpha_{1,1}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1 \oplus A_3$

$$\mathbf{n}=10, \text{ degeneration} \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{10,2}$
orbits of	$(\alpha_3, \alpha_5, \alpha_4, \alpha_7)$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=10, \text{ degeneration } \left( \begin{array}{cccc} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{array} \right) \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{10,2}$
orbits of	$(\alpha_{1,4}, \alpha_{1,3}, \alpha_{2,3}, \alpha_{3,3})$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=10, \text{ degeneration } \left( \begin{array}{cccc} 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{array} \right) \subset 8\mathbb{A}_2:$$

$j$	18	22 *
$H$	$H_{10,1}$	$H_{10,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,1}, \alpha_{1,3}, \alpha_{2,3})$	$(\alpha_{1,3}, \alpha_{2,3}, \alpha_{1,5}, \alpha_{2,5})$
$(S_{N_j}^\perp)^{(2)}$	$A_3$	$2A_2$

$$\mathbf{n}=10, \text{ degeneration } \left( \begin{array}{cccc} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & & 2\mathbb{A}_2 \end{array} \right) \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2:$$

$j$	22 *
$H$	$H_{10,1}$
orbits of	$(\alpha_{1,3}, \alpha_{1,5}, \alpha_{2,5}, \alpha_{1,9}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_2$

## 5 Tables of types of degenerations of Kählerian K3 surfaces with symplectic automorphism group $(C_2)^3$

Here, we consider the case of  $(C_2)^3$ . This case contains much less cases, and it is much easier than the previous case of  $D_8$ .



Table 6: Types and lattices  $S$  of degenerations of codimension  $\geq 2$  of Kählerian K3 surfaces with symplectic automorphism group  $(C_2)^3$ .

$\mathbf{n}$	$ G $	$G$	$Deg$	$\text{rk } S$	$qs$
9	8	$(C_2)^3$	$((2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1)_I$	16	$2_{II}^{+2}, 4_0^{+4} *$
			$((2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1)_{II}$	16	$2_{II}^{+4}, 4_{II}^{+2} *$
			$(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$	16	$2_{II}^{+4}, 4_7^{+1}, 8_1^{+1} *$
			$(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3$	16	$2_{II}^{+4}, 4_{II}^{+2} *$
			$(2\mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$	16	$2_{II}^{-4}, 4_4^{-2} *$
			$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	16	$2_{II}^{+4}, 4_{II}^{+2}$
			$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$	16	$2_{II}^{+6} *$
			$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$	16	$2_{II}^{+6} *$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I \\ & & 2\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1$	17	$4_7^{+5} *$
			$((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	17	$2_{II}^{+2}, 4_6^{+2}, 8_1^{+1} *$
			$((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	17	$2_{II}^{+4}, 8_7^{+1} *$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_3$	17	$2_{II}^{+2}, 4_7^{+3} *$
			$((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$	17	$2_{II}^{+2}, 4_7^{+3} *$
			$(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 10\mathbb{A}_1$	17	$2_{II}^{+2}, 4_7^{+3} *$
			$(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_1$	17	$2_{II}^{+4}, 8_7^{+1} *$
			$(2\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	17	$2_{II}^{+4}, 4_7^{+1} *$
			$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$	17	$2_{II}^{+4}, 8_7^{+1}$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I \\ & & & 2\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$	18	$4_6^{+4} *$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	18	$4_5^{+3}, 8_1^{+1} *$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 2\mathbb{A}_3$	18	$4_6^{+4} *$

$\mathbf{n}$	$G$	$Deg$	$\text{rk } S$	$q_s$
9	$(C_2)^3$	$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1$	18	$4_6^{+4}$
		$((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$4_6^{+4} *$
		$((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$2_{II}^{-2}, 4_2^{-2} *$
		$\begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 6\mathbb{A}_1$	18	$2_{II}^{-2}, 4_1^{+1}, 8_5^{-1} *$
		$\begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3$	18	$2_{II}^{-2}, 4_2^{-2} *$
		$((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$2_{II}^{-2}, 4_2^{-2} *$
		$(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 14\mathbb{A}_1$	18	$2_{II}^{+2}, 4_5^{-1}, 8_5^{-1} *$
		$(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 8\mathbb{A}_1$	18	$2_{II}^{+2}, 4_6^{+2} *$
		$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1$	18	$2_{II}^{+2}, 4_6^{+2} *$
		$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & & 8\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1$	19	$4_5^{+3} *$
		$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 6\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1 \amalg 2\mathbb{A}_3$	19	$4_4^{-2}, 8_5^{-1} *$
		$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$4_5^{+3} *$

<b>n</b>	$G$	$Deg$	$\text{rk } S$	$q_S$
9	$(C_2)^3$	$\left( \begin{array}{ccccc} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & & 8\mathbb{A}_1 \end{array} \right) \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$4_5^{+3} *$
		$((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1} *$
		$\left( \begin{array}{ccccc} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & & 4\mathbb{A}_1 \end{array} \right) \subset 4\mathbb{A}_3 \amalg 4\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1} *$

Table 7: Markings  $S \subset N_j$  by Niemeier lattices, and lattices  $S_{N_j}^\perp$  of degenerations of codimension  $\geq 2$  of Kählerian K3 surfaces with symplectic automorphism group  $(C_2)^3$  in notations [18], [19], [20].

**n=9**, degeneration  $((2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1)_I$ :

$j$	21	21	21	23	23 *
$H$	$H_{9,1}$	$H_{9,2}$	$H_{9,2}$	$H_{9,3}$	$H_{9,4}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}), \dots$	$(\alpha_{2,2}, \alpha_{2,3}), \dots$	$(\alpha_{2,2}, \alpha_{1,1}), \dots$	$(\alpha_6, \alpha_8), \dots$	$(\alpha_1, \alpha_8), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$4A_1$	$2A_1$	$2A_1$	$\{0\}$

**n=9**, degeneration  $((2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1)_{II}$ :

$j$	23 *
$H$	$H_{9,1}$
orbits of	$(\alpha_5, \alpha_6)$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

**n=9**, degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$ :

$j$	21	21	21	23	23 *
$H$	$H_{9,1}$	$H_{9,2}$	$H_{9,2}$	$H_{9,1}$	$H_{9,4}$
orbits of	$(\alpha_{1,1}, \alpha_{2,3}), \dots$	$(\alpha_{2,3}, \alpha_{1,2}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}), \dots$	$(\alpha_5, \alpha_2), \dots$	$(\alpha_1, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$6A_1$	$4A_1$	$2A_1$	$4A_1$	$\{0\}$

$\mathbf{n}=9$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=9$ , degeneration  $(2\mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$ :

$j$	21	23 *
$H$	$H_{9,1}$	$H_{9,3}$
orbits of	$(\alpha_{1,1}, \alpha_{1,3}), \dots$	$(\alpha_6, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$6A_1$	$2A_1$

$\mathbf{n}=9$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	21	23
$H$	$H_{9,2}$	$H_{9,1}$
orbits of	$(\alpha_{1,2}, \alpha_{1,3}), \dots$	$(\alpha_2, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$4A_1$

$\mathbf{n}=9$ , degeneration  $(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{9,1}$
orbits of	$(\alpha_{2,3}, \alpha_{1,3})$
$(S_{N_j}^\perp)^{(2)}$	$8A_1$

$\mathbf{n}=9$ , degeneration  $(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{9,2}$
orbits of	$(\alpha_2, \alpha_4)$
$(S_{N_j}^\perp)^{(2)}$	$8A_1$

$\mathbf{n}=9$ , degeneration  $\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I \\ & & 2\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1$ :

$j$	21	21	23	23 *
$H$	$H_{9,1}$	$H_{9,2}$	$H_{9,3}$	$H_{9,4}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,4}), \dots$	$(\alpha_{2,2}, \alpha_{2,3}, \alpha_{1,1}), \dots$	$(\alpha_6, \alpha_8, \alpha_{10}), \dots$	$(\alpha_1, \alpha_8, \alpha_{10}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$2A_1$	$\{0\}$

$\mathbf{n}=9$ , degeneration  $((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	21	21	21	23 *
$H$	$H_{9,1}$	$H_{9,2}$	$H_{9,2}$	$H_{9,4}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,3}), \dots$	$(\alpha_{2,3}, \alpha_{2,7}, \alpha_{1,2}), \dots$	$(\alpha_{2,3}, \alpha_{1,1}, \alpha_{1,2}), \dots$	$(\alpha_1, \alpha_8, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$4A_1$	$2A_1$	$\{0\}$

$\mathbf{n}=9$ , degeneration  $((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{9,1}$
orbits of	$(\alpha_5, \alpha_6, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=9$ , degeneration  $\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_3$ :

$j$	21 *	21
$H$	$H_{9,2}$	$H_{9,2}$
orbits of	$(\alpha_{2,3}, \alpha_{2,2}, \alpha_{1,2}), \dots$	$(\alpha_{1,1}, \alpha_{2,2}, \alpha_{1,2}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$2A_1$

$\mathbf{n}=9$ , degeneration  $((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	21	23 *
$H$	$H_{9,1}$	$H_{9,3}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3}), \dots$	$(\alpha_6, \alpha_8, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$2A_1$

$\mathbf{n}=9$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 10\mathbb{A}_1$ :

$j$	21	21 *	23
$H$	$H_{9,2}$	$H_{9,2}$	$H_{9,1}$
orbits of	$(\alpha_{2,7}, \alpha_{1,2}, \alpha_{1,3}), \dots$	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3}), \dots$	$(\alpha_5, \alpha_2, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$2A_1$	$4A_1$

$\mathbf{n}=9$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_1$ :

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=9$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{9,1}$
orbits of	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$6A_1$

$\mathbf{n}=9$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	21	23
$H$	$H_{9,2}$	$H_{9,1}$
orbits of	$(\alpha_{1,2}, \alpha_{1,3}, \alpha_{1,7}), \dots$	$(\alpha_2, \alpha_3, \alpha_4), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$4A_1$

$\mathbf{n}=9$ , degeneration  $\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I \\ & & & 2\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{9,3}$
orbits of	$(\alpha_6, \alpha_8, \alpha_{10}, \alpha_{16}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=9, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1:$$

$j$	21	21	23 *
$H$	$H_{9,1}$	$H_{9,2}$	$H_{9,4}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,4}, \alpha_{2,3}), \dots$	$(\alpha_{2,3}, \alpha_{2,7}, \alpha_{1,1}, \alpha_{1,2}), \dots$	$(\alpha_1, \alpha_8, \alpha_{10}, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$	$\{0\}$

$$\mathbf{n}=9, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 2\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{1,1}, \alpha_{2,3}, \alpha_{2,2}, \alpha_{1,2}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=9, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1:$$

$j$	21	23
$H$	$H_{9,1}$	$H_{9,3}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,4}, \alpha_{1,3}), \dots$	$(\alpha_6, \alpha_8, \alpha_{10}, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$2A_1$

$$\mathbf{n}=9, \text{ degeneration } ((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1:$$

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{2,7}, \alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$\mathbf{n}=9$ , degeneration  $((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{9,1}$
orbits of	$(\alpha_5, \alpha_6, \alpha_2, \alpha_3), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=9, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 6\mathbb{A}_1:$$

$j$	21 *	21
$H$	$H_{9,2}$	$H_{9,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,7}, \alpha_{1,3}), \dots$	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{1,1}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$	$2A_1$

$$\mathbf{n}=9, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,3}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=9$ , degeneration  $((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3$ :

$j$	21 *
$H$	$H_{9,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,3}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=9$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 14\mathbb{A}_1$ :



$j$	21	23 *
$H$	$H_{9,2}$	$H_{9,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3}, \alpha_{1,7}), \dots$	$(\alpha_5, \alpha_2, \alpha_3, \alpha_4), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$	$4A_1$

$\mathbf{n}=9$ , degeneration  $(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 8\mathbb{A}_1$ :

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{1,3}, \alpha_{1,7}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$\mathbf{n}=9$ , degeneration  $(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1$ :

$j$	23 *
$H$	$H_{9,1}$
orbits of	$(\alpha_2, \alpha_3, \alpha_4, \alpha_7), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=9, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & & 8\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{9,3}$
orbits of	$(\alpha_6, \alpha_8, \alpha_{10}, \alpha_{16}, \alpha_2), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=9, \text{ degeneration } \begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 6\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1 \amalg 2\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{1,1}, \alpha_{2,7}, \alpha_{1,3}, \alpha_{2,2}, \alpha_{1,2}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=9, \text{ degeneration} \left( \begin{array}{ccccc} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & & 4\mathbb{A}_1 \end{array} \right) \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{1,1}, \alpha_{2,2}, \alpha_{1,2}, \alpha_{2,3}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=9, \text{ degeneration} \left( \begin{array}{ccccc} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & & 8\mathbb{A}_1 \end{array} \right) \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_3:$$

$j$	21 *
$H$	$H_{9,1}$
orbits of	$(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,4}, \alpha_{2,3}, \alpha_{1,3}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$2A_1$

$$\mathbf{n}=9, \text{ degeneration } ((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1:$$

$j$	23 *
$H$	$H_{9,1}$
orbits of	$(\alpha_5, \alpha_6, \alpha_2, \alpha_3, \alpha_4), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

$$\mathbf{n}=9, \text{ degeneration} \left( \begin{array}{ccccc} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & & 4\mathbb{A}_1 \end{array} \right) \subset 4\mathbb{A}_3 \amalg 4\mathbb{A}_1:$$

$j$	21 *
$H$	$H_{9,2}$
orbits of	$(\alpha_{2,2}, \alpha_{1,2}, \alpha_{2,3}, \alpha_{1,3}, \alpha_{1,7}), \dots$
$(S_{N_j}^\perp)^{(2)}$	$4A_1$

## 6 Final remarks

We hope to give more details and applications in further variants of the paper and further publications.

We hope to consider remaining groups  $D_6$ ,  $C_4$ ,  $(C_2)^2$ ,  $C_3$ ,  $C_2$  later as well.

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